



Институт физики микроструктур
Российской академии наук

Плазмоны в твердом теле

Рудаков Артур

Мнс, аспирант 1 года обучения

15.12.2022

План семинара

- Теоретическое введение
- Объемные плазмоны
- Поверхностные плазмоны
- Двумерные плазмоны

Теоретическое введение

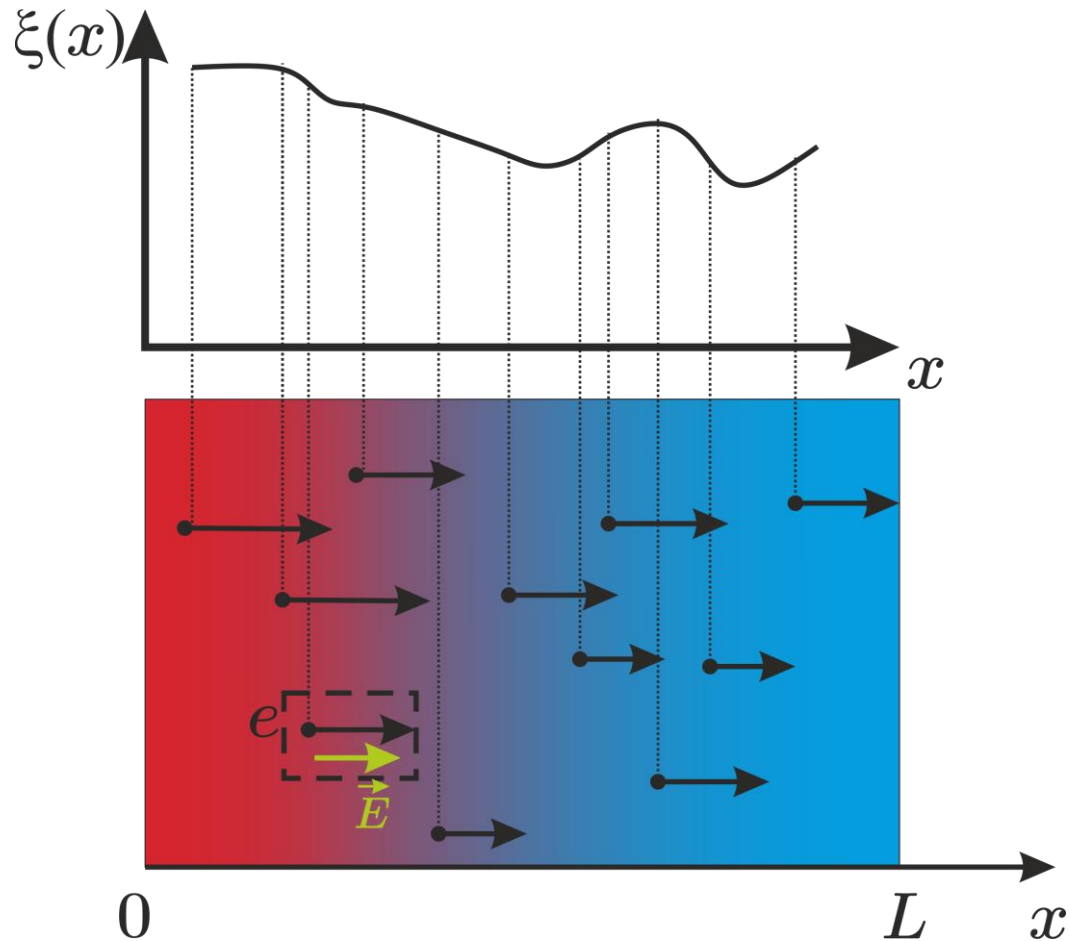
$$\xi = \xi(x) \quad \delta n = n \frac{\partial \xi}{\partial x} \text{ в случае, если } \frac{\partial \xi}{\partial x} \ll 1$$

Из уравнения: $\frac{\partial E}{\partial x} = \frac{4\pi e \delta n}{\kappa}$ получаем

$$E(x) = \frac{4\pi e n \xi(x)}{\kappa}, F = -eE$$

$$m \ddot{\xi} = - \frac{4\pi e^2 n}{\kappa} \xi$$

$$\omega_p = \sqrt{\frac{4\pi e^2 n}{m \kappa}} - \text{плазменная частота}$$



Закон дисперсии плазменных волн в объемных материалах

Пусть в среде имеется плотность тока $\mathbf{j}$, обусловленная колебанием заряженных частиц. Если длина волны плазмона существенно больше длины свободного пробега электрона, то выполняется

$$\mathbf{j}(\mathbf{r}, \omega) = \sigma(\omega)\mathbf{E}(\mathbf{r}, \omega)$$
$$\operatorname{div}\mathbf{D} = 0; \quad \operatorname{div}\mathbf{H} = 0; \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}; \quad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t};$$

$$\operatorname{rot}(\operatorname{rot}\mathbf{E}) = \operatorname{grad}(\operatorname{div}\mathbf{E}) - \nabla^2\mathbf{E} = \frac{\omega^2\kappa_L}{c^2}\left(-\frac{4\pi\sigma}{i\omega\kappa_L} + 1\right)\mathbf{E}, \quad \nabla^2\mathbf{E} + \frac{\omega^2\kappa_L}{c^2}\left(-\frac{4\pi\sigma}{i\omega\kappa_L} + 1\right)\mathbf{E} = 0$$

Волна продольная, то $\mathbf{q}||\mathbf{E}$, $\operatorname{rot}\mathbf{E} = i[\mathbf{qE}] = 0$. Следовательно, $1 - \frac{4\pi\sigma}{i\omega\kappa_L} = 0$. Пользуясь формулой Друде при $\tau \rightarrow \infty$, получаем, $1 - \frac{\omega_p^2}{\omega^2} = 0$. Таким образом, спектр объемных плазмонов имеет вид $\omega = \omega_p$

$$\omega^2 \approx \omega_p^2 + \frac{3k_B T}{m\kappa_L} q^2 + \dots$$

Плазменные волны в металлах

Эксперимент по возбуждению объемных плазмонов был впервые проведен Рутеманом и Лангом в 1948 году.

Теоретическое объяснение было предложено Бомом и Пайнсом в 1953 году

Diskrete Energieverluste mittelschneller Elektronen beim Durchgang durch dünne Folien¹⁾

Von Gerhard Ruthemann

(Der Fakultät für Naturwissenschaften und Ergänzungsfächer der Technischen Hochschule Danzig eingereichte Dissertation, 1. Teil)

(Mit 20 Abbildungen)

PHYSICAL REVIEW

VOLUME 92, NUMBER 3

NOVEMBER 1, 1953

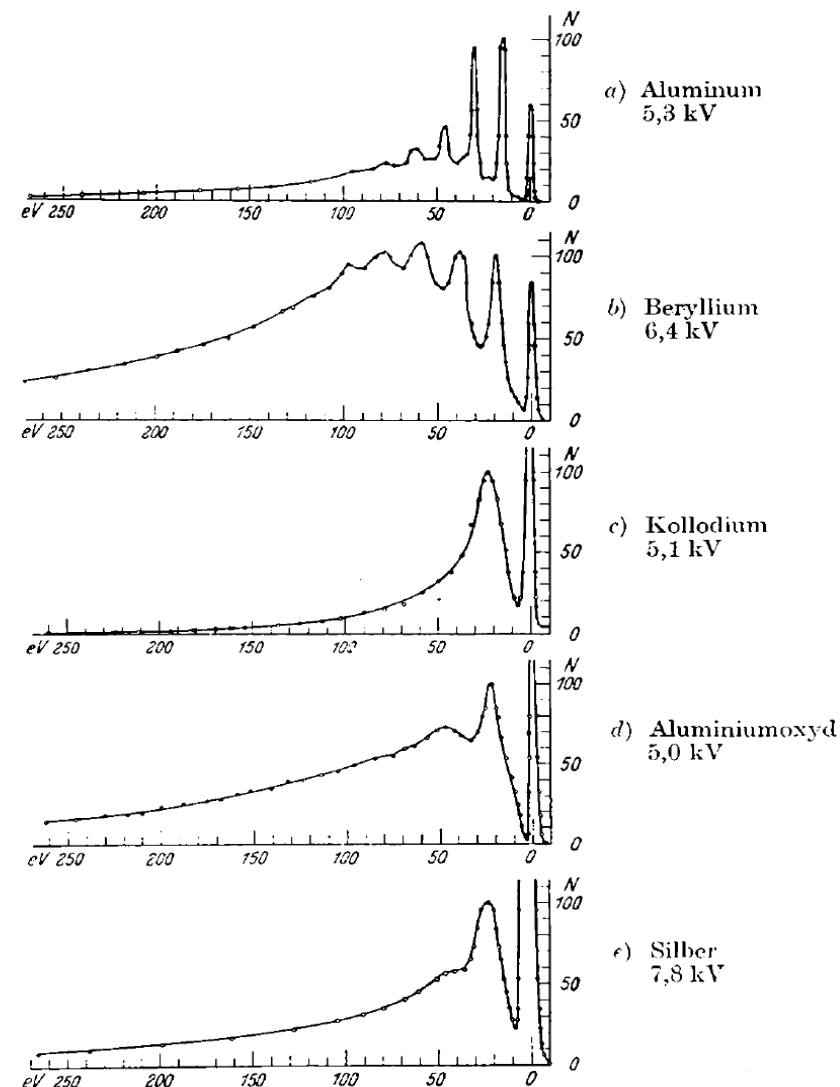
A Collective Description of Electron Interactions: III. Coulomb Interactions in a Degenerate Electron Gas

DAVID BOHM, *Faculdade de Filosofia, Ciências e Letras, Universidade de São Paulo, São Paulo, Brazil*

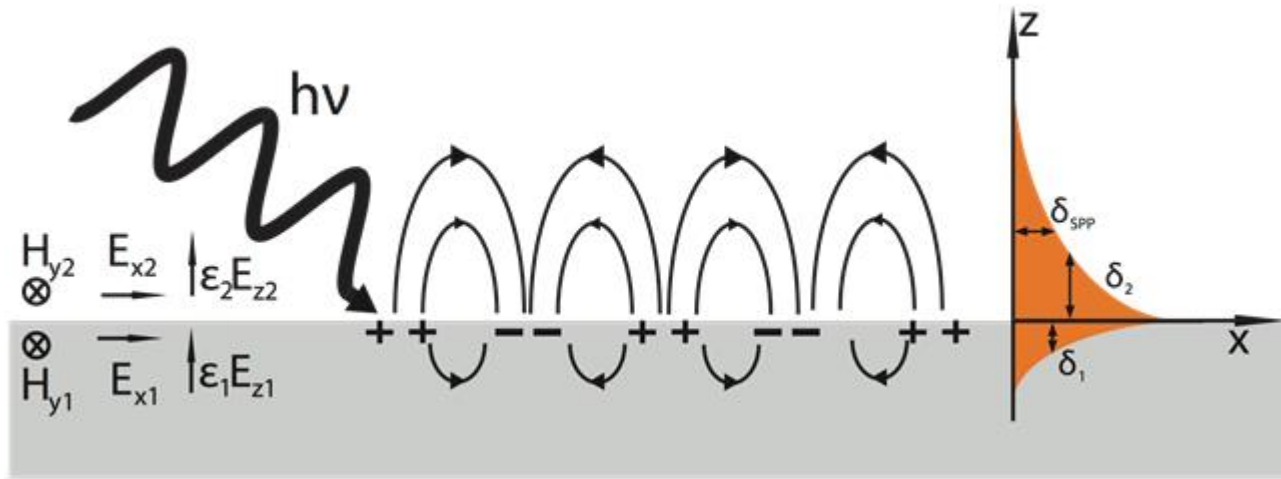
AND

DAVID PINES, *Department of Physics, University of Illinois, Urbana, Illinois*

(Received May 21, 1953)



Поверхностные плазмоны



Sommerfeld, Arnold. "Ueber die Fortpflanzung elektrodynamischer Wellen längs eines Drahtes." *Annalen der Physik* 303.2 (1899): 233-290.

$$\frac{\partial}{\partial z} \left(\frac{1}{\kappa(\omega, z)} \frac{\partial H_y}{\partial z} \right) - \left(\frac{q^2}{\kappa(\omega, z)} - \frac{\omega^2}{c^2} \right) H_y = 0.$$

$$H_y^{met}(z) = H_0 \exp \left(-z \sqrt{q^2 + \frac{\omega^2}{c^2} |\kappa_1|} \right),$$

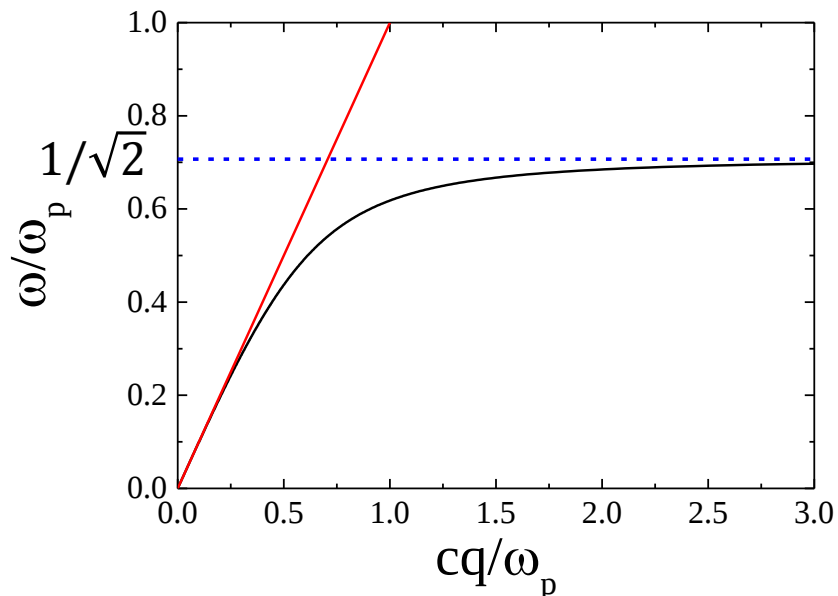
$$H_y^{diel}(z) = H_0 \exp \left(z \sqrt{q^2 - \frac{\omega^2}{c^2} \kappa_2} \right)$$

Граничное условие: $H_y|_{0-\epsilon} = H_y|_{0+\epsilon}$

$$\frac{1}{\kappa_2} \frac{\partial H_y}{\partial z} \Big|_{0-\epsilon} = - \frac{1}{|\kappa_1|} \frac{\partial H_y}{\partial z} \Big|_{0+\epsilon}$$

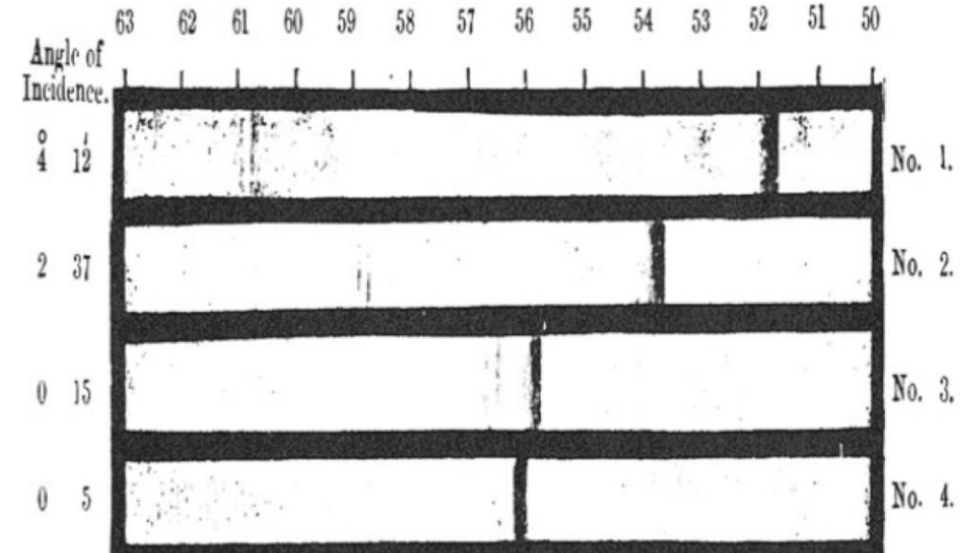
$$q^2 = \frac{\omega^2}{c^2} \frac{|\kappa_1| \kappa_2}{|\kappa_1| - \kappa_2}$$

Спектр поверхностных плазмонов



$$q^2 = \frac{\omega^2}{c^2} \frac{|\kappa_1|}{|\kappa_1| - 1}$$

$$\kappa_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$



1. Wood, R. W. (1902). XLII. On a remarkable case of uneven distribution of light in a diffraction grating spectrum. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 4(21), 396-402.
2. Fano, Ugo. "The theory of anomalous diffraction gratings and of quasi-stationary waves on metallic surfaces (Sommerfeld's waves)." *JOSA* 31.3 (1941): 213-222.

Возбуждение поверхностных плазмонов (конфигурация Кретчмана)

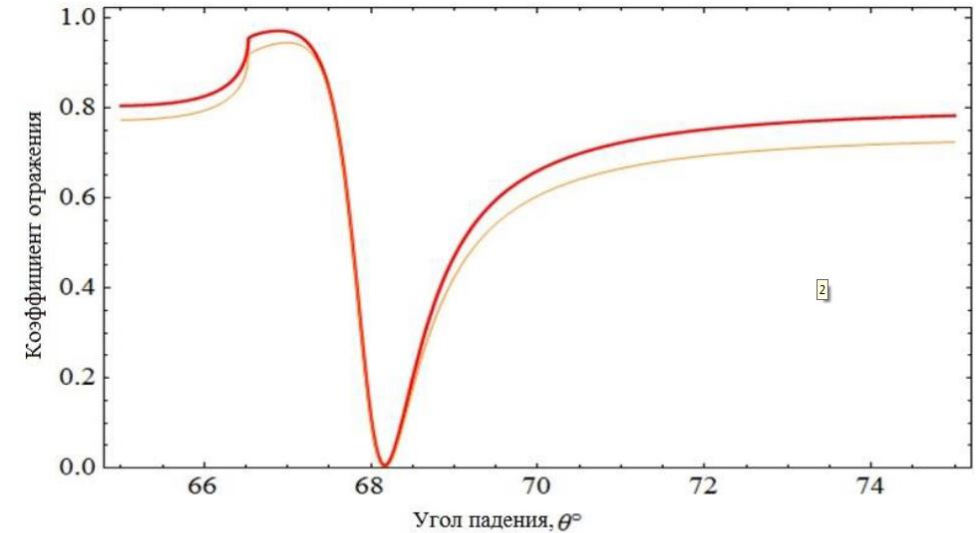
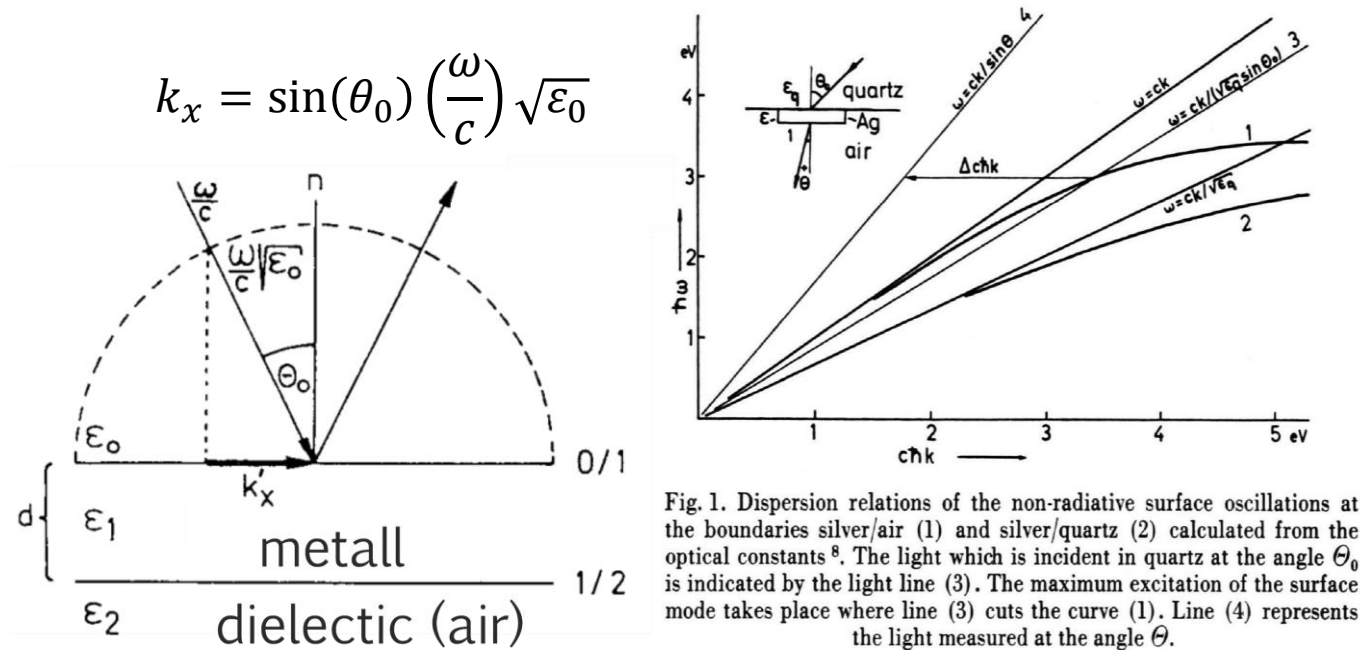


Рисунок 1.3 – Характерная зависимость коэффициента отражения от угла падения для конфигурации Кретчмана [34].

Kretschmann E, Raether H. Zeitschrift für Naturforschung A. 1968 Dec 1;23(12):2135-6.

Возбуждение поверхностных плазмонов (конфигурация Отто)

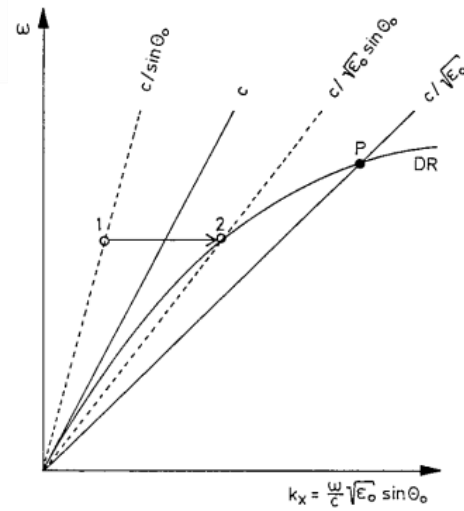
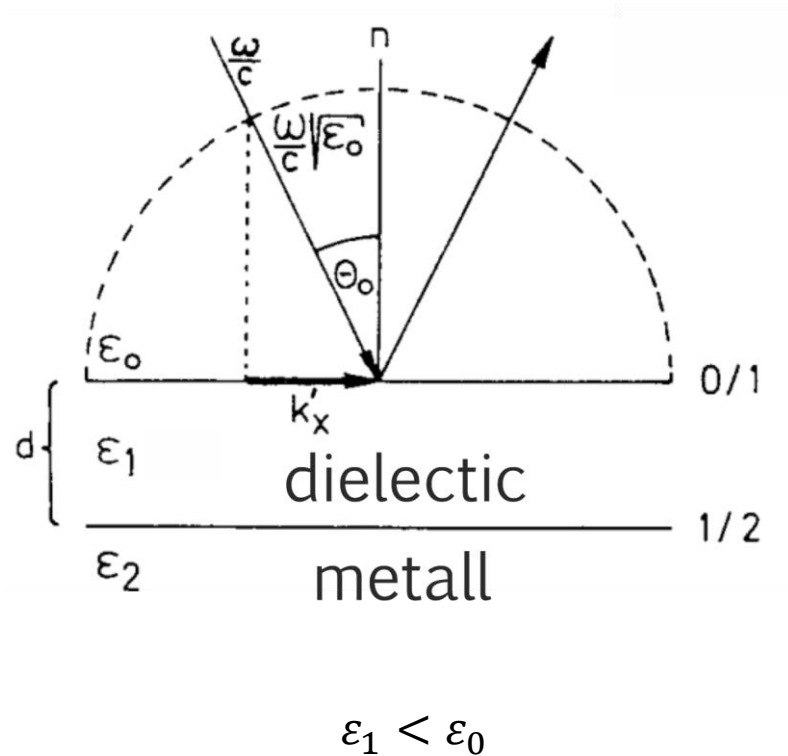


Fig.2.7. The ATR method: Dispersion relation DR of SPs for a quartz/metal/air system $\varepsilon_2 = 1$, c : light line in vacuum, $c/\sqrt{\varepsilon_0}$: light line in the medium ε_0 . Since the light line $c/\sqrt{\varepsilon_0}$ lies to the right of the DR up to a certain k_x , light can excite SPs of frequencies ω below the crossing point P on the metal/air side. The SPs on the interface 0/1 metal/quartz cannot be excited, since their DR lies to the right of $c/\sqrt{\varepsilon_0}$ because $\varepsilon_0 > 1$

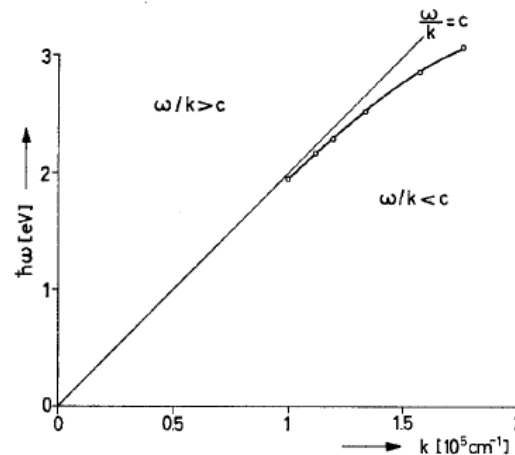


Fig. 6. Experimental dispersion $\omega(k)$ of the nonradiative SPW at the silver air-interface and $\omega = kc$ for plane waves in air

A. Otto. Zeitschrift für Physik A Hadrons and nuclei. – 1968. – V. 216. – N 4. – P. 398-410.

Возбуждение поверхностных плазмонов (использование решетки)

$$q_{SP} = k_0 \sqrt{\kappa_{diel}} \sin(\theta) + n \frac{2\pi}{\Lambda}$$

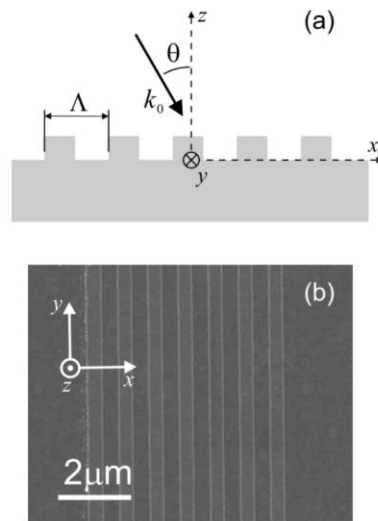
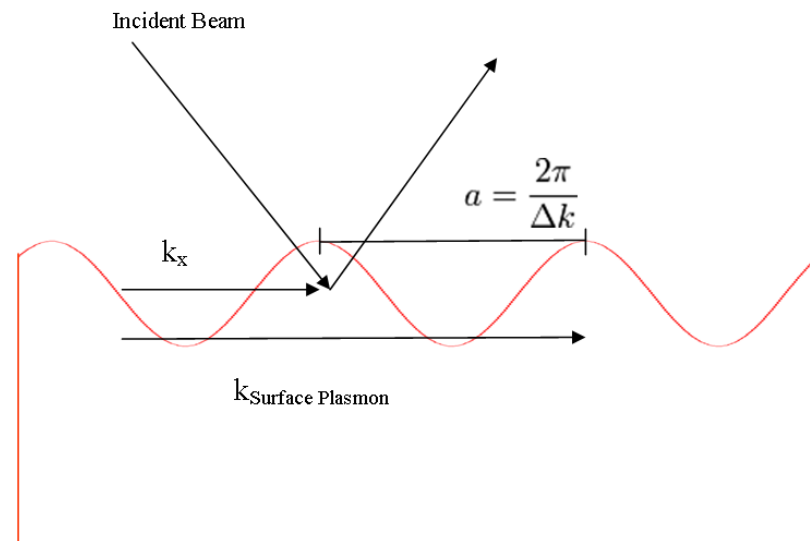


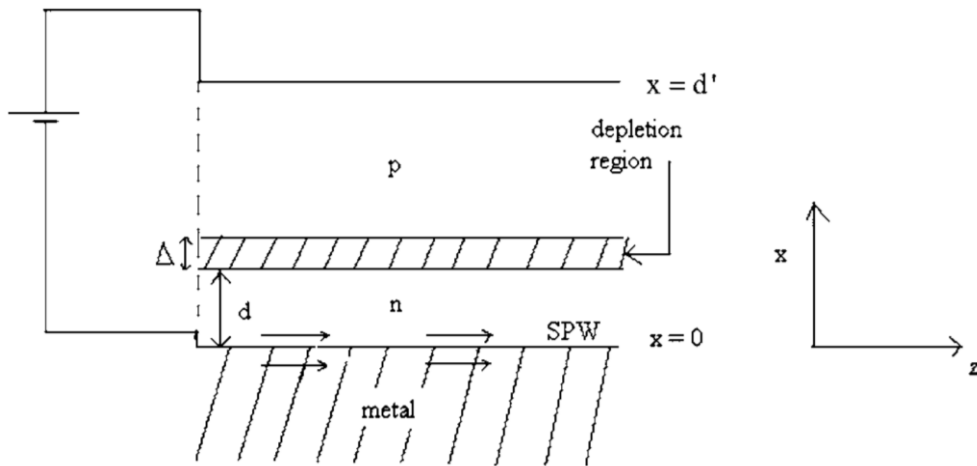
FIG. 1. (a) Geometry of the configuration under study and coordinate system. (b) Scanning electron microscope image of a typical structure under investigation. In this case it is a set of seven periodically arranged ridges. Period is 800 nm.



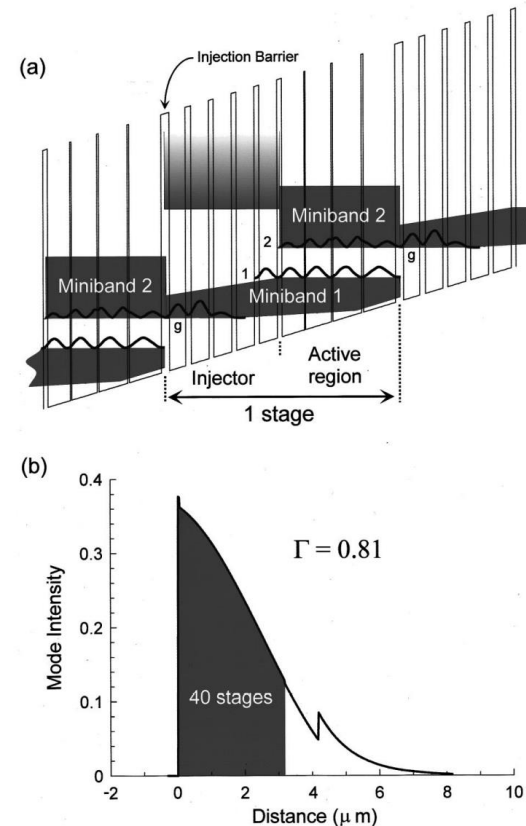
Radko, I. P., Bozhevolnyi, S. I., Brucoli, G., Martín-Moreno, L., García-Vidal, F. J., & Boltasseva, A. (2008). Efficiency of local surface plasmon polariton excitation on ridges. *Physical Review B*, 78(11), 115115.

Лазеры на поверхностных плазмонах

Поверхностные плазмонные моды, заключенные на границе раздела между металлом и полупроводником, используются вместо обычных диэлектрических волноводов в ККЛ

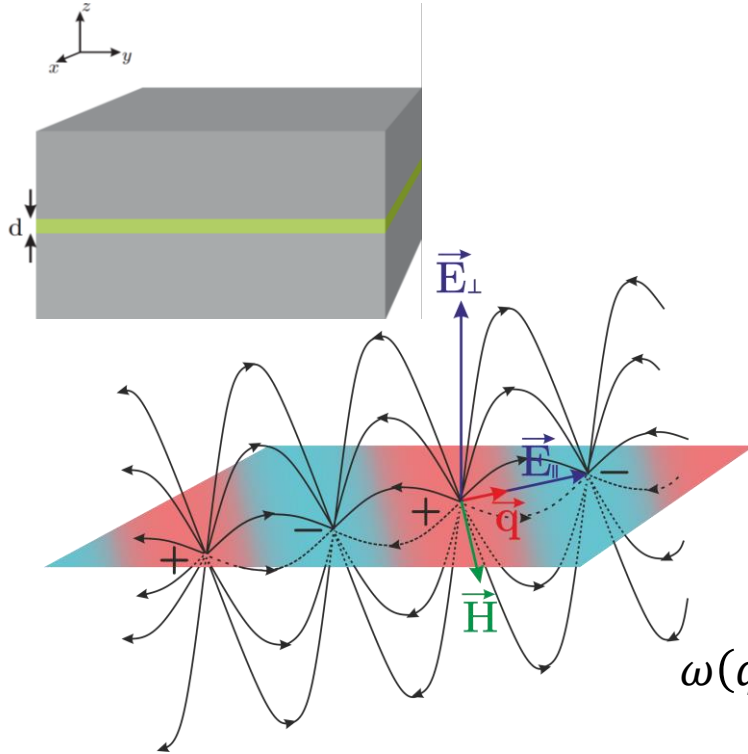


1. Kumar P, Tripathi VK, Liu CS. Journal of Applied Physics. 2008;104(3):033306.
2. Tredicucci A, Gmachl C, Wanke MC, Capasso F, Hutchinson AL, Sivco DL, Chu SN, Cho AY. Applied Physics Letters. 2000; 77(15):2286-8.



Двумерные плазмоны

$$\sigma_{2d}(\omega, z) = \sigma(\omega)\delta(z)$$



$$\omega(q) = \sqrt{\frac{2\pi e^2 n_s}{m\kappa}} q$$

$$-i \frac{\partial}{\partial z} \left(\frac{\kappa(z)}{Q^2(z)} \frac{\partial E_{||}}{\partial z} \right) = \frac{4\pi\sigma(\omega)\delta(z)E_{||}}{\omega} - i\kappa(z)E_{||}$$

$$E_{||}(z) = E_0 \exp(-Qz)$$

$$1 - \frac{2\pi\sigma(\omega)}{i\omega\kappa} \sqrt{q^2 - \frac{\omega^2}{c^2}\kappa} = 0 \quad \sigma(\omega) = i \frac{e^2 n_s}{\omega m}$$

$$q^2 = \frac{\omega^2}{c^2}\kappa + \left(\frac{\kappa m \omega^2}{2\pi e^2 n_s} \right)^2$$

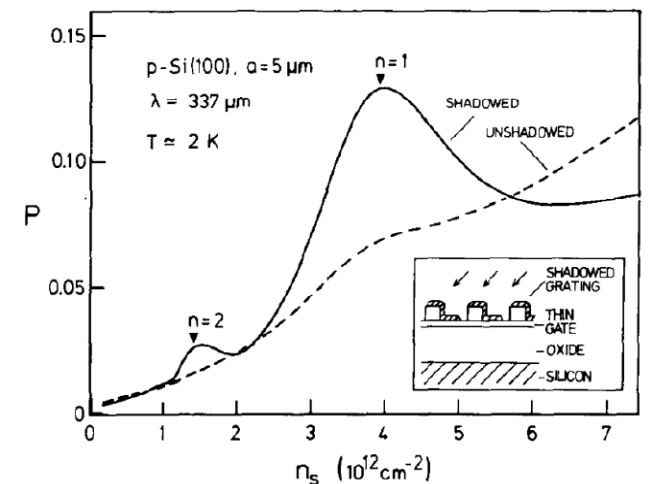
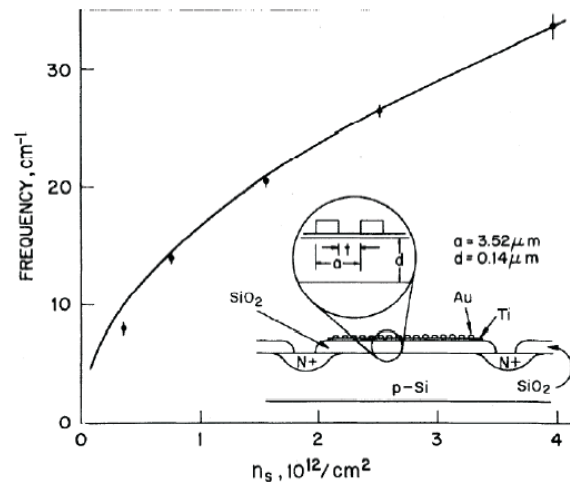
Плазмон хорошо определен при $\omega > \frac{2\pi e^2 n_s}{mc\sqrt{\kappa}}$

$$Q = \sqrt{q^2 - \frac{\omega^2}{c^2}\kappa}$$

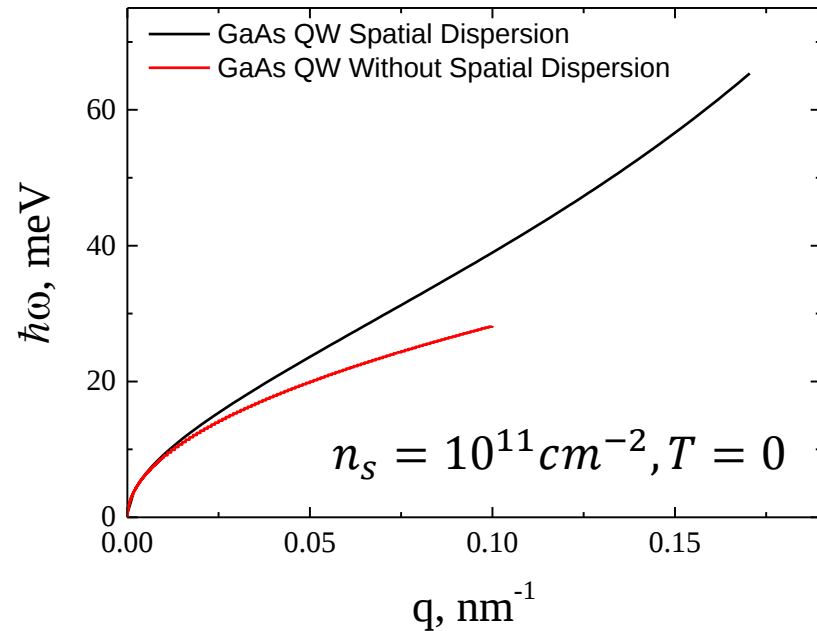
Уравнение имеет решение в случае, $\frac{c}{\sqrt{\kappa}} > \frac{\omega}{q} = v_{ph}$

1. Allen S. J., Tsui D. C., Logan R. A. // Phys. Rev. Lett. — 1977

2. Theis T., Kotthaus J., Stiles P. // Surface Science. — 1978. — V. 73. — p. 434—436.



Учет пространственной дисперсии поляризуемости электронного газа

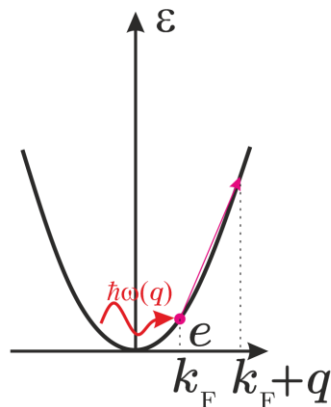


$$\chi(\omega) = -\frac{\sigma(\omega)}{i\omega}$$

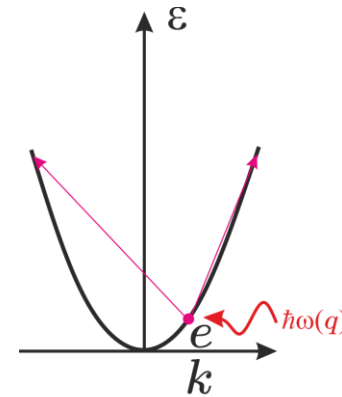
$$\chi(\omega, \mathbf{q}) = \frac{e^2}{q^2 S} \sum_{\mathbf{k}} \frac{[f(\mathbf{k}) - f(\mathbf{k} + \mathbf{q})]}{\epsilon(\mathbf{k} + \mathbf{q}) - \epsilon(\mathbf{k}) - \hbar\omega(\mathbf{q}) - i\hbar\alpha} \quad \alpha \rightarrow 0$$

Затухание Ландау

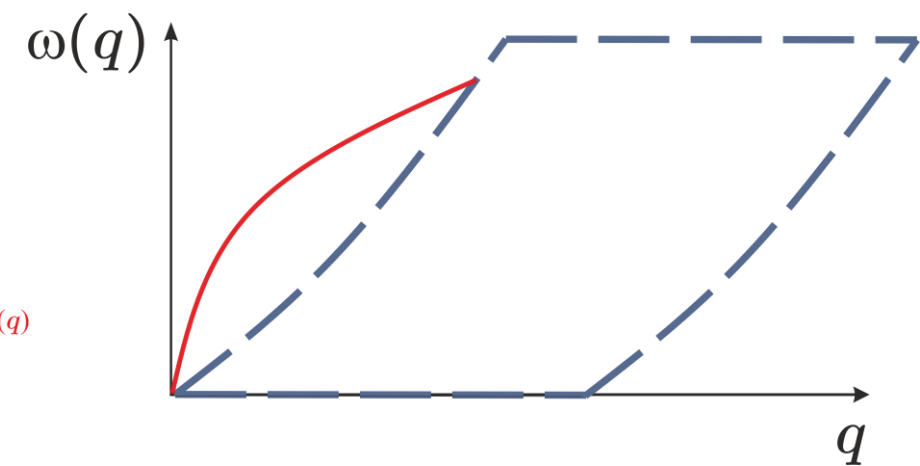
$$\frac{\hbar^2 k_F^2}{2m} + \hbar\omega(q) = \frac{\hbar^2 (k_F + q)^2}{2m}$$



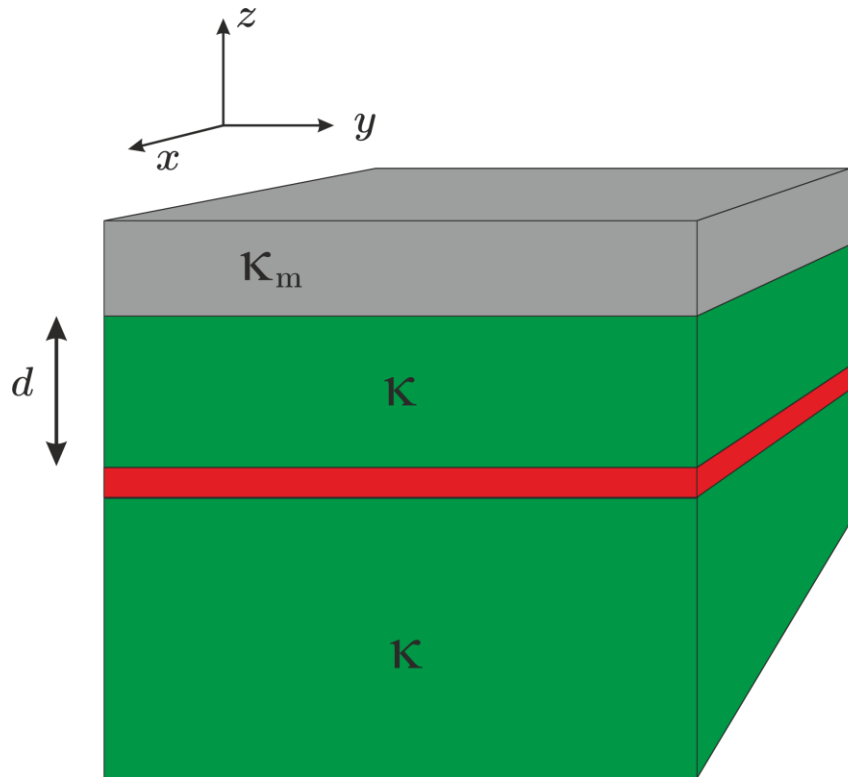
Landau damping



Drude losses



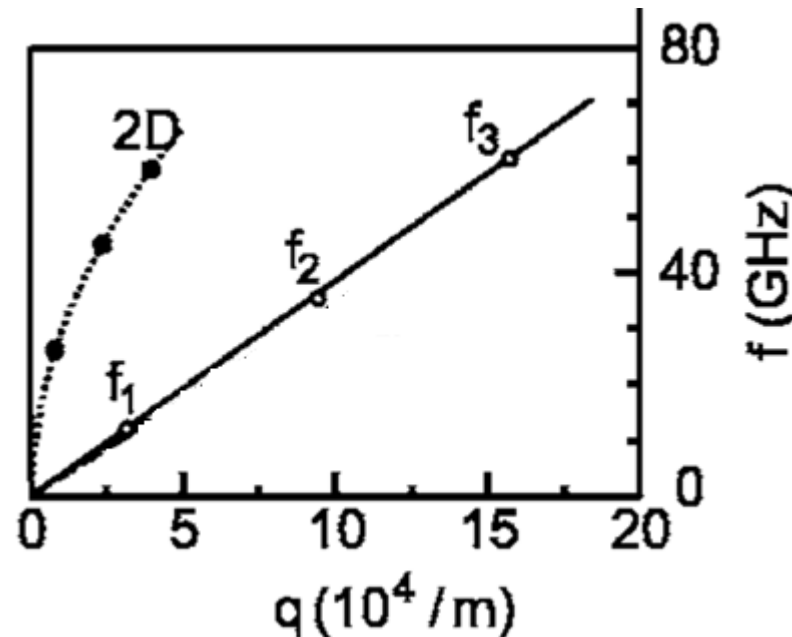
Экранированные плазмоны



$$qd \ll 1 \quad |\kappa_m| \gg \kappa$$

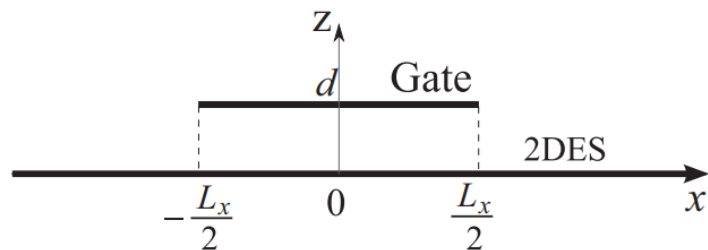
$$1 + i \frac{4\pi\sigma(\omega)}{\kappa\omega} = -\frac{1}{qd}$$

$$\omega(q) = q \sqrt{\frac{4\pi e^2 n_s d}{m\kappa}}$$



1. A. V. Chaplik, Zh. Eksp. Teor. Fiz. 62, 746 1972 Sov. Phys. JETP 35
2. Muravev, V. M., et al. *Physical Review B* 75.19 (2007): 193307.

«Проксимити»-плазмоны



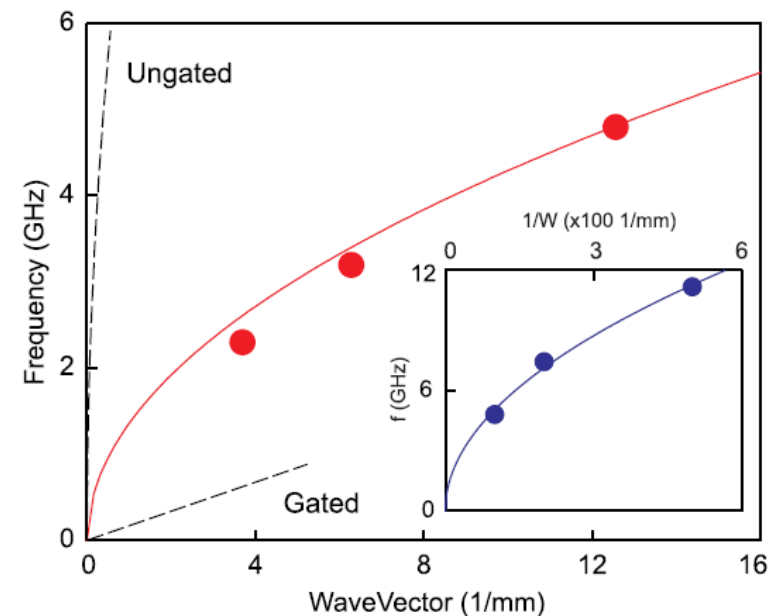
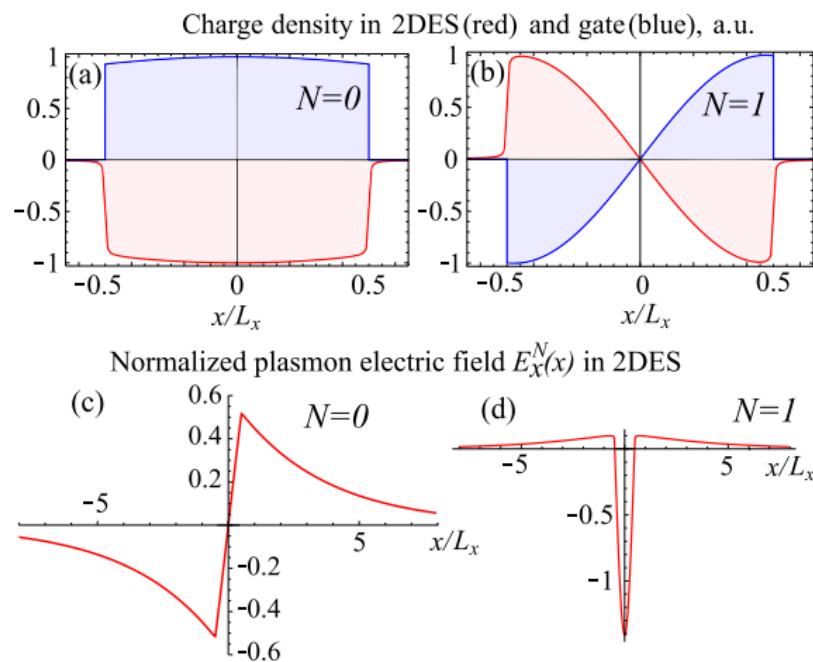
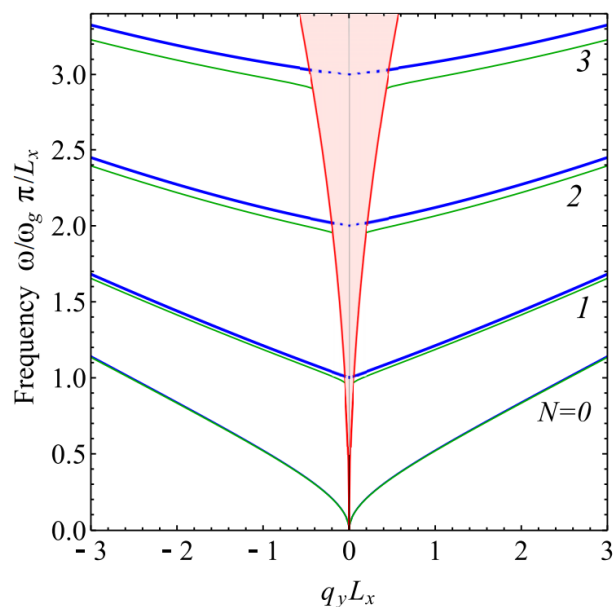
$$k \left(\tan \frac{kL_x}{2} \right)^{\pm 1} = \pm |q_y|,$$

$$k^2 = \frac{\omega^2}{\frac{4\pi n_s e^2 d}{m\kappa}} - q_y^2$$

$$q_y L_x \ll 1,$$

$$\omega(q_y) = \sqrt{\frac{8\pi n_s e^2 d}{m\kappa} \frac{|q_y|}{L_x}}, \quad \text{для } N = 0$$

FIG. 1. System under consideration with the gate and the 2DES assumed to be infinite in y.



1. Zabolotnykh A. A., Volkov V. A. // Phys. Rev. B. — 2019. — V. 99. — p. 165304.
2. Muravev V. M., Gusikhin P. A., Zarezin A. M., Andreev I. V., Gubarev S. I. Phys. Rev. B. — 2019. — V. 99,. — p. 241406.

Генерация двумерных плазмонов в полевых транзисторах

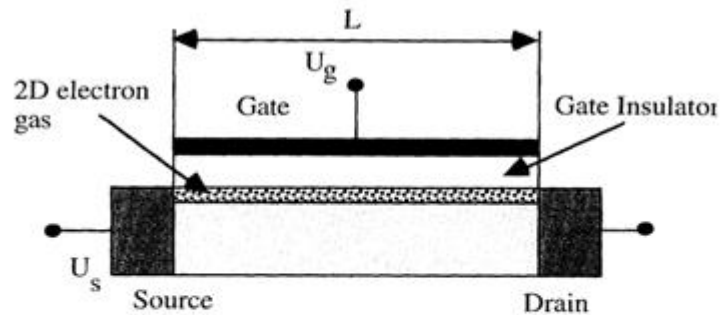


FIG. 1. Schematic structure of the ballistic FET. The gate length L is much smaller than the mean free path λ but much longer than the mean free path for electron-electron collisions, λ_{ee} .

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{e}{m} \frac{\partial U}{\partial x}$$

$$\frac{\partial U}{\partial t} + \frac{\partial(Uv)}{\partial x} = 0$$

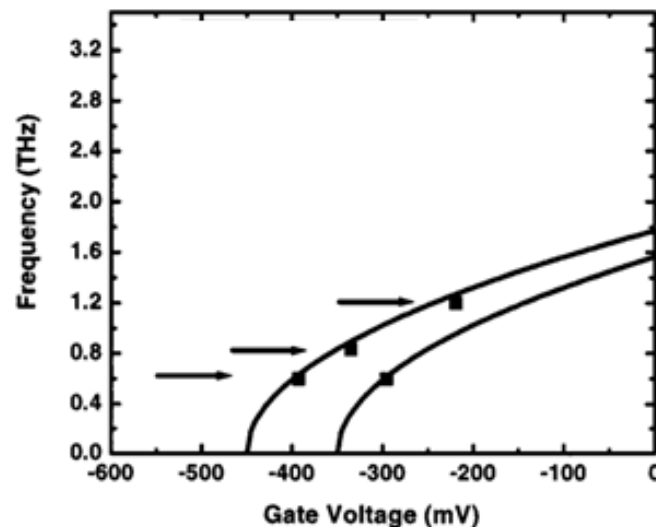
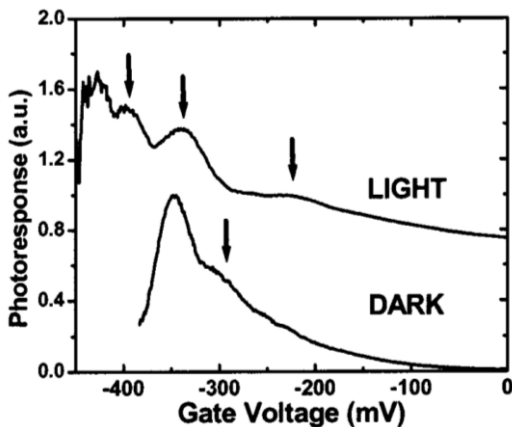
$$j = env_0$$

$$k = \frac{\omega}{v_0 + s} \quad s = \sqrt{\frac{eU_0}{m}}$$

$$v(t) = v_0 + v_1 \exp(-i\omega t), \quad U(t) = U_0 + U_1 \exp(-i\omega t)$$

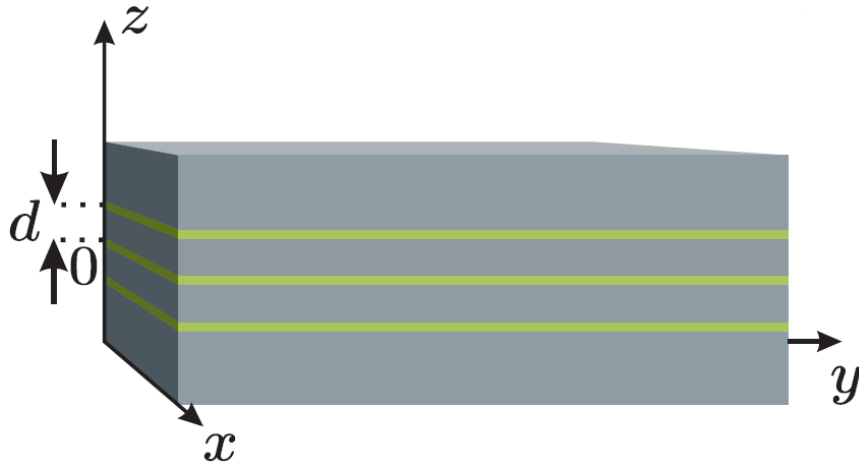
$$U(0) = 0, \quad U_0 v_1(L) + v_0 U_1(L) = 0$$

$$\omega' = \frac{|s^2 - v_0^2|}{2Ls} \pi n \quad \omega'' = \frac{s^2 - v_0^2}{2Ls} \ln \frac{s + v_0}{s - v_0} \quad |v_0| < s$$

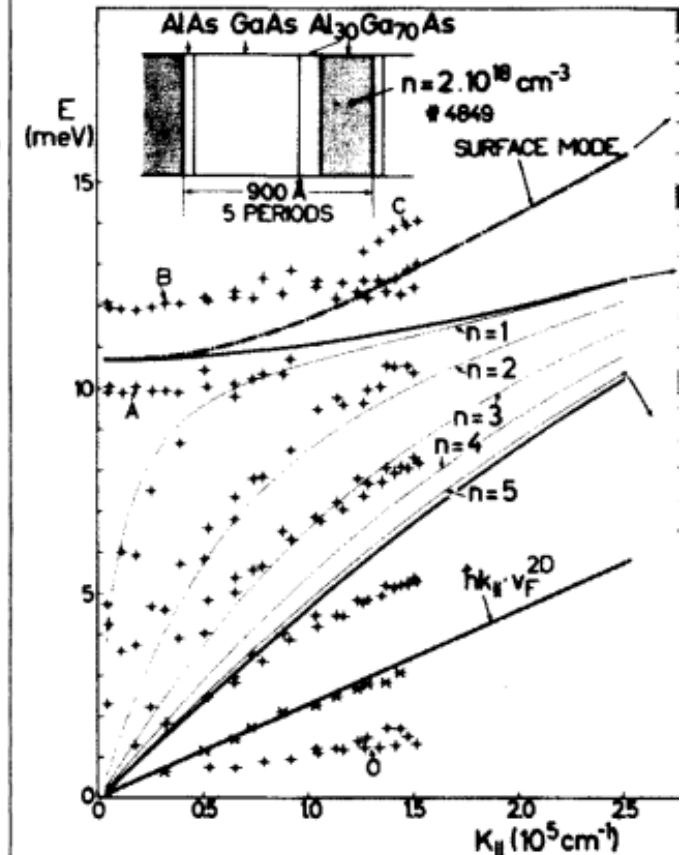
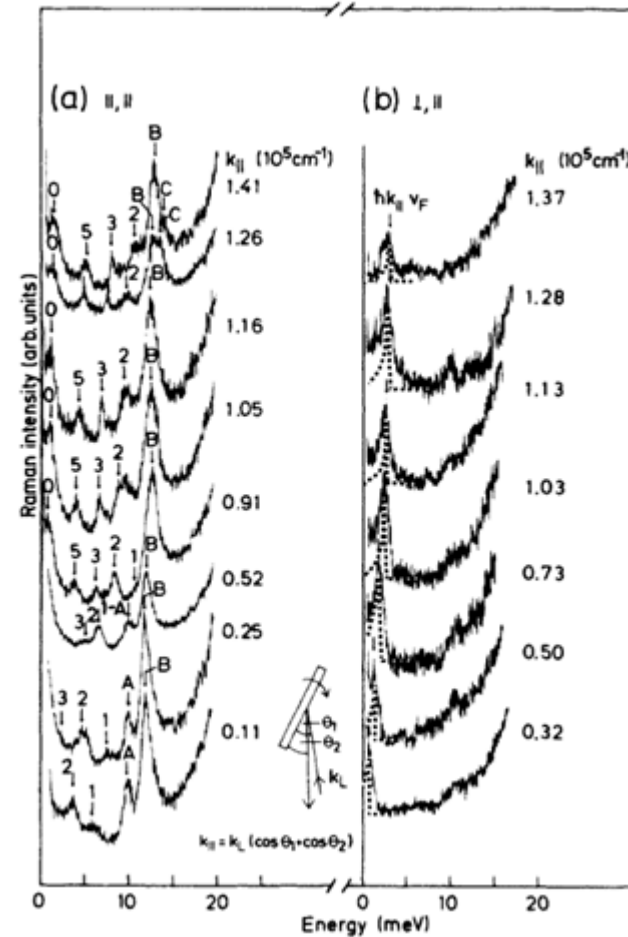


1. Dyakonov M, Shur M. Physical review letters. 1993; 71(15):2465.
2. Knap W, Deng Y, Romyantsev S, Shur M. Applied physics letters. 2002; 81(24):4637-9.

Плазмоны в многослойных структурах



1. Fasol G, Mestres N, Hughes HP, Fischer A, Ploog K. *Physical review letters*. 1986; 56(23):2517.
2. Jain, Jainendra K., and Philip B. Allen. *Physical review letters* 54.22 (1985): 2437.



Плазмоны в графене

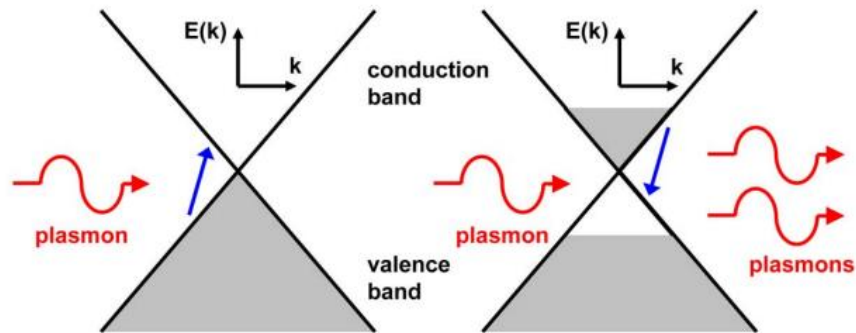


Fig. 1. (Left) Energy bands of graphene showing stimulated absorption of plasmons. (Right) Population inverted graphene bands showing stimulated emission of plasmons.

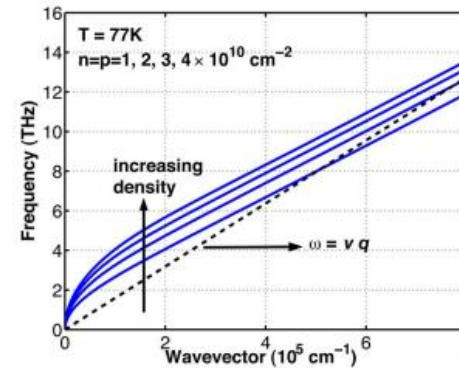
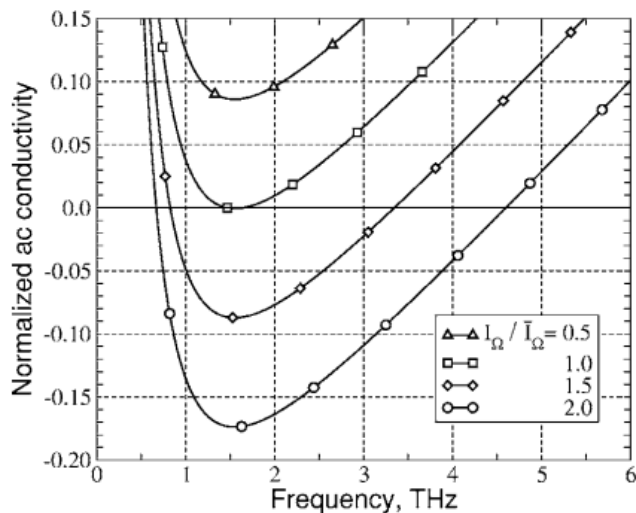


Fig. 3. Calculated plasmon dispersion relation in graphene at 77 K is plotted for different electron-hole densities ($n = p = 1, 2, 3, 4 \times 10^{10} \text{ cm}^{-2}$). The condition $\omega(q) > vq$ is satisfied for frequencies that have net gain in the terahertz range. The assumed values of v and τ are 10^8 cm/s and 0.5 ps , respectively.

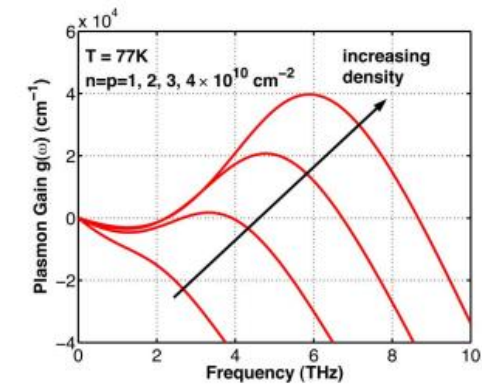
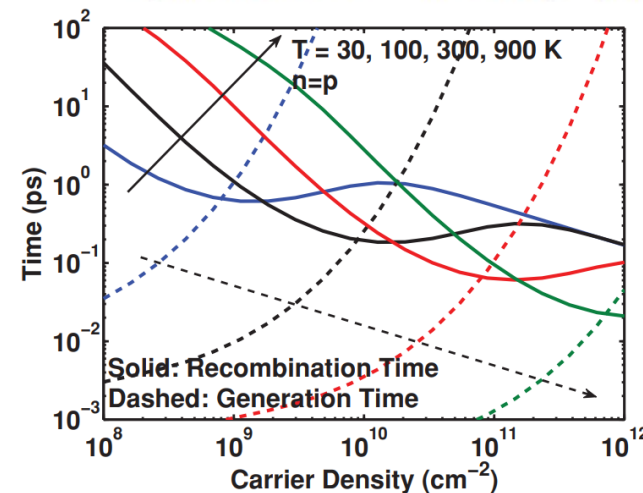
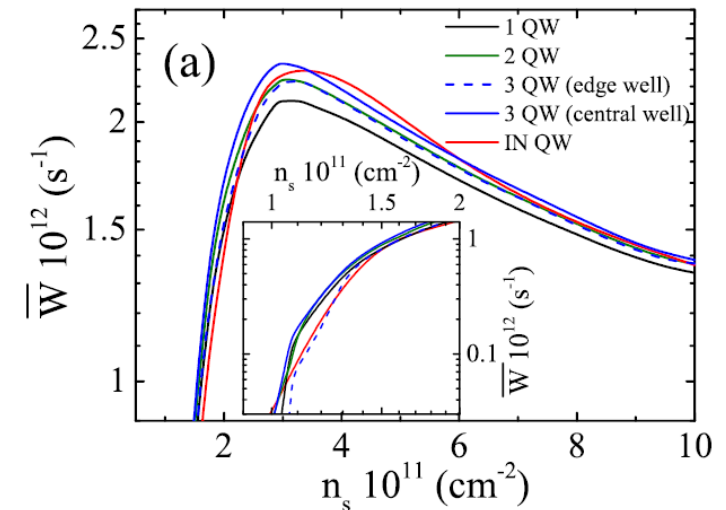
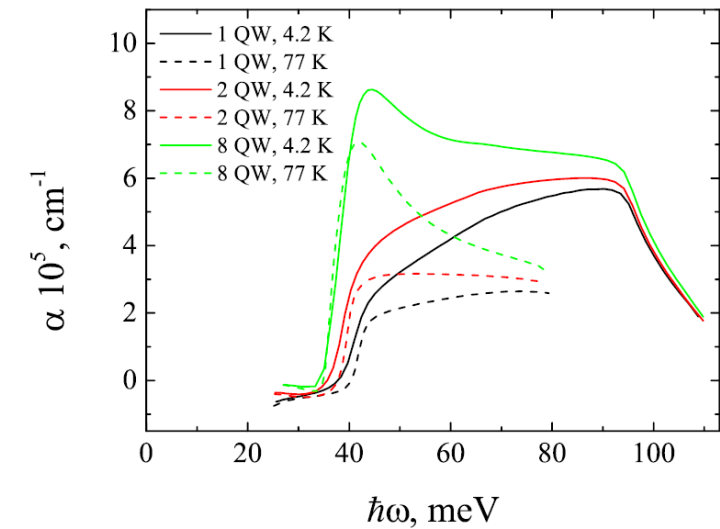
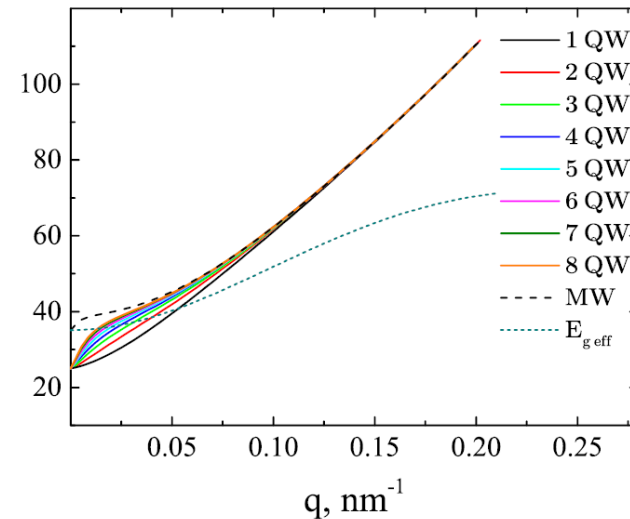
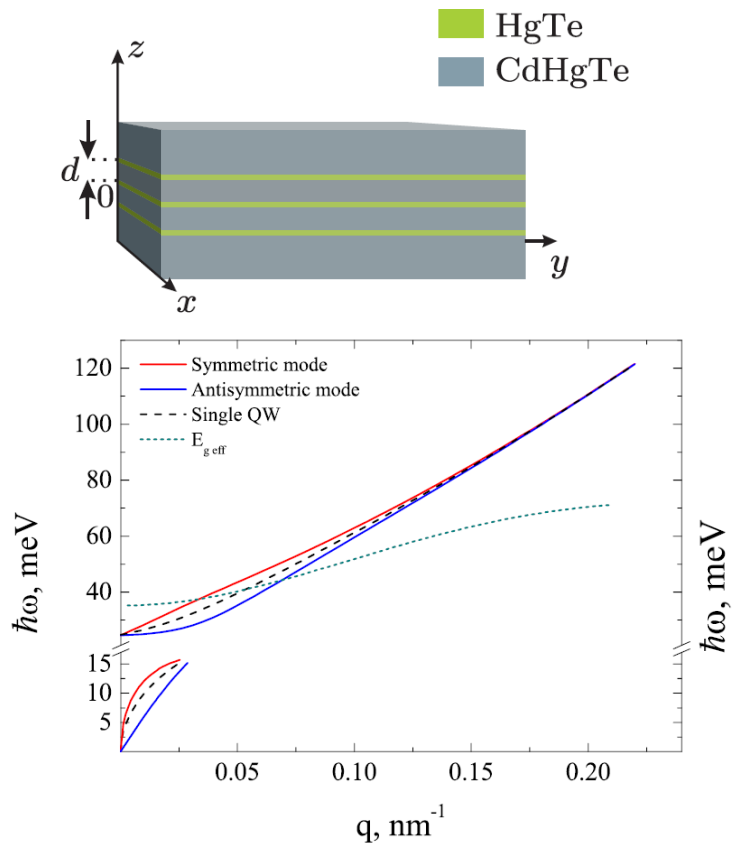


Fig. 4. Net plasmon gain (interband gain minus intraband loss) in graphene at 77 K is plotted for different electron-hole densities ($n = p = 1, 2, 3, 4 \times 10^{10} \text{ cm}^{-2}$). The assumed values of v and τ are 10^8 cm/s and 0.5 ps , respectively.



1. Rana F. IEEE Transactions on Nanotechnology. 2008;7(1):91-9.
2. Rana, Farhan, et al. *Physical Review B* 84.4 (2011): 045437.
3. Ryzhii V, Ryzhii M, Otsuji T. Journal of Applied Physics. 2007;101(8):083114.

Плазмоны в гетероструктурах CdHgTe/HgTe



1. Rudakov, A. O., V. Ya Aleshkin, and V. I. Gavrilenko. *Journal of Optics* 24.7 (2022): 075001.
2. Aleshkin, V. Ya, and A. O. Rudakov. *Physical Review B* 106.16 (2022): 165307.