

# Структура электронных мод в майорановских нанопроводах

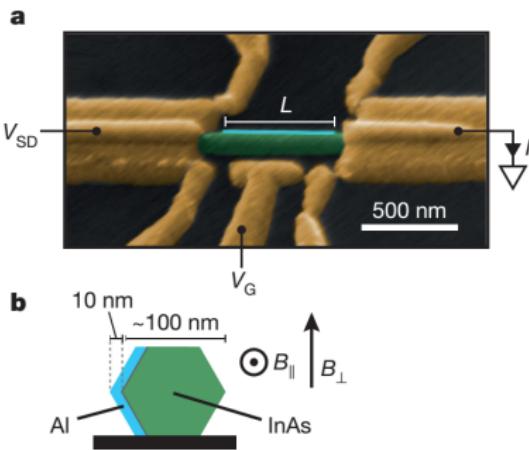
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15 февраля 2018 г.

1. A.E. Antipov, A. Bargerbos, G.W. Winkler, B. Bauer, E. Rossi, R.M. Lutchyn, *Effects of gate-induced electric fields on semiconductor Majorana nanowires*, arXiv:1801.02616, 8 Jan 2018.
2. A.E.G. Mikkelsen, P. Kotetes, P. Krogstrup, K. Flensberg, *Hybridization at superconductor-semiconductor interfaces in Majorana devices*, arXiv:1801.03439, 10 Jan 2018.

1. Контакт Al/InAs
2. Распределение электронной плотности (подход Томаса - Ферми)
3. Распределение электронной плотности (самосогласованный подход)
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# Exponential protection of zero modes in Majorana islands<sup>1</sup>



**Figure 1 | Majorana island device.** **a**, Electron micrograph (false colour) of a device that is lithographically similar to the measured devices. Yellow, Ti/Au contacts; green, InAs nanowire; light blue, two-facet Al shell (of length  $L$ );  $V_{SD}$ , applied voltage bias;  $I$ , measured current;  $V_G$ , gate voltage. **b**, Cross-section of a hexagonal InAs nanowire showing the orientation of the Al shell and field directions  $B_{\parallel}$  and  $B_{\perp}$ .

<sup>1</sup>S. M. Albrecht, A. P. Higginbotham, M. Madsen, F. Kuemmeth, T. S. Jespersen, J. Nygård, P. Krogstrup, and C. M. Marcus, *Nature* **531**, 206–209 (2016).

# 1. Контакт Al/InAs

Parameter	InAs	Al
$m^*$	0.026 [94]	1
$\epsilon_r$	15.15	
$W$ , eV	-0.25	
$g^{\text{bare}}$	-15 [95]	2
$\alpha$ , eV· nm	0.01 [96]	0
$\varepsilon_F$ , eV	0	11.27 [97]
$\Delta_0$ , meV	0	0.34 [97]
$L_z$ , nm	60	10
$L_y$ , nm	52	52

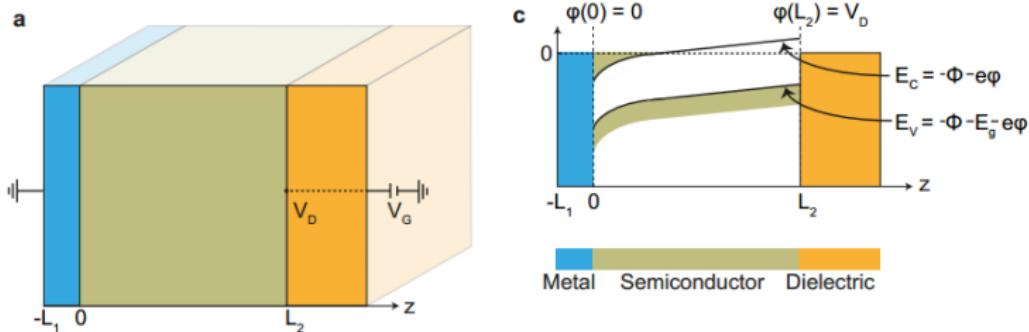
TABLE I. Physical parameters for InAs and Al.

## 2. Подход Томаса - Ферми (формулировка)

Уравнение на электростатическое поле:

$$\frac{d}{dz} \left[ \epsilon_r \frac{d\phi}{dz} \right] = - \frac{\rho(z)}{\epsilon_0}. \quad (1)$$

Границные условия:  $\phi(0) = 0$ ,  $\phi(L_2) = V_D$ .



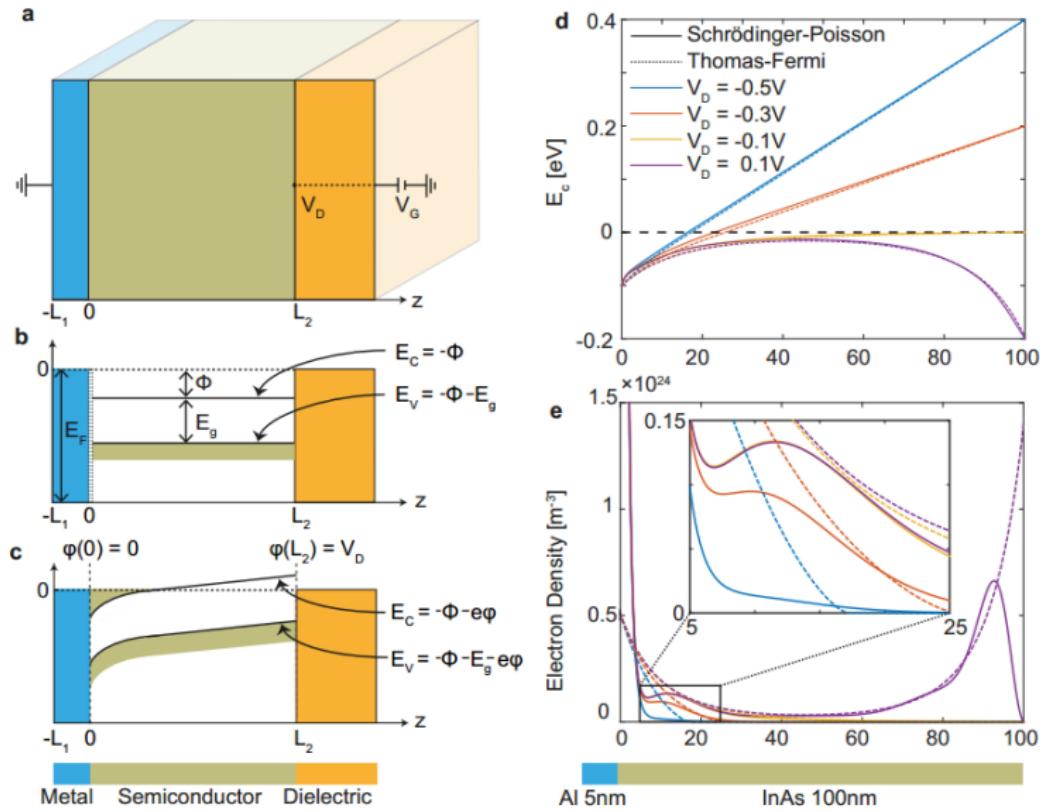
Распределение электронной плотности:

$$\rho(z) = -\frac{e}{3\pi^2} \left[ \sqrt{2m_{InAs}\epsilon_F(z)/\hbar} \right]^3. \quad (2)$$

$$\ell_{TF}^2 \frac{d^2\varphi}{dz^2} = - [1 + \varphi(z)]^{3/2}. \quad (3)$$

Здесь  $\epsilon_F(z) = \Phi + e\phi(z)$ ,  $\ell_{TF}^{-1} = \sqrt{e|\rho(\phi=0)|/\epsilon_{InAs}\Phi}$  и  $\varphi = e\phi/\Phi$ .

## 2. Подход Томаса - Ферми (результаты)



### 3. Самосогласованный подход (формулировка)

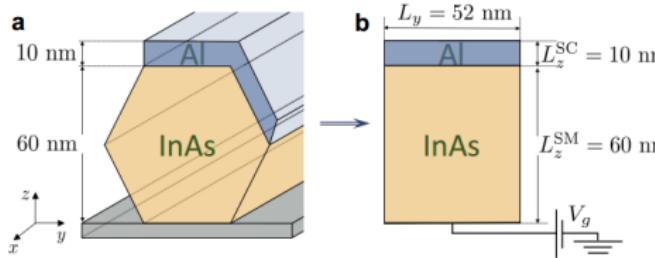


FIG. 1. (a) SM-SC heterostructure based on hexagonal nanowire. 10 nm thick Al layer (blue) is deposited on 2 facets of InAs (brown) hexagonal wire with a height of 60 nm. The back gate is shown schematically in gray. (b) Rectangular geometry of the wire that supports the same number of subbands. The back gate is emulated by a boundary condition at the bottom.

Самосогласованное решение уравнений Пуассона и задачи на собственные значения:

$$\hat{H}_n = -\partial_z \left[ \frac{1}{2m^*(z)} \partial_z \right] + \frac{1}{2m^*(z)} \left( \hat{k}_x^2 + \hat{k}_y^2 \right) + \phi(z) - \varepsilon_F(z) - \alpha(z) \left( \hat{k}_x \sigma_y - \hat{k}_y \sigma_x \right) + \frac{\mu_B g(z) B}{2} \sigma_x . \quad (4)$$

Плотность заряда:

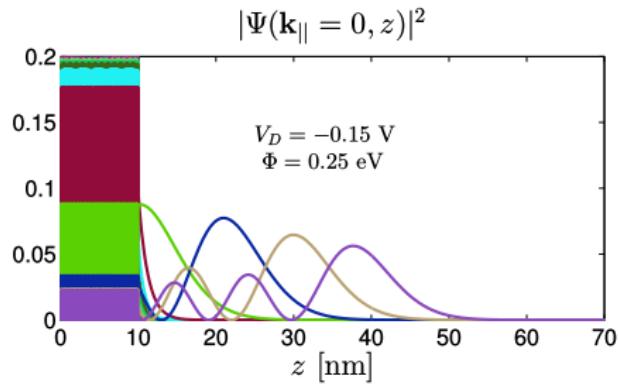
$$\rho(z) = \frac{1}{2\pi L_y} \sum_{n_y, E[\Psi] \leq 0} \int dk_x |\Psi_{k_x, n_y}(z)|^2 . \quad (5)$$

### 3. Самосогласованный подход (разностная схема)

В полупроводнике  $\Delta z \approx 2\text{\AA}$ , в металлической пленке  $\Delta z \approx 0.1\text{\AA}$ ,

$$\partial_z \left[ \frac{1}{2m^*(z)} \partial_z \right] \psi(z) \rightarrow \frac{1}{z_{i+1} - z_{i-1}} \left( \frac{1}{m_{i+1/2}^*} \frac{\psi_{i+1} - \psi_i}{z_{i+1} - z_i} - \frac{1}{m_{i-1/2}^*} \frac{\psi_i - \psi_{i-1}}{z_i - z_{i-1}} \right), \quad (6)$$

где  $m_{i\pm 1/2}^* = (m_i^* \pm m_{i\pm 1}^*)/2$ .



Для уравнения Пуассона

$$\phi_{\text{in}}^i(z) = \lambda \phi_{\text{out}}^{i-1}(z) + (1 - \lambda) \phi_{\text{in}}^{i-1}(z). \quad (7)$$

### 3. Самосогласованный подход (результаты)

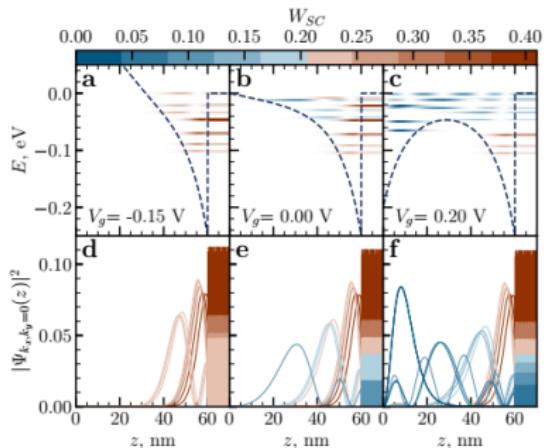


FIG. 5. Eigenstates of the Hamiltonian (4) at  $k_x = 0$  for  $V_g = -0.15 \text{ V}$ ,  $V_g = 0 \text{ V}$ , and  $V_g = 0.2 \text{ V}$ . The top panel shows the electrostatic potential for reference purposes, with horizontal lines denoting the bottom of each band below the Fermi energy. The color scale indicates the weight in the superconductor, and in the semiconducting part, the intensity indicates the square modulus of the eigenstate. The lower panel shows the eigenfunctions explicitly with the same color coding.

$$W_{SC} = 1 - \sum_{n_y, \sigma} \int_0^{L_z^{\text{SM}}} |\Psi(k_F)|^2 dz . \quad (8)$$

### 3. Самосогласованный подход (реальная геометрия)

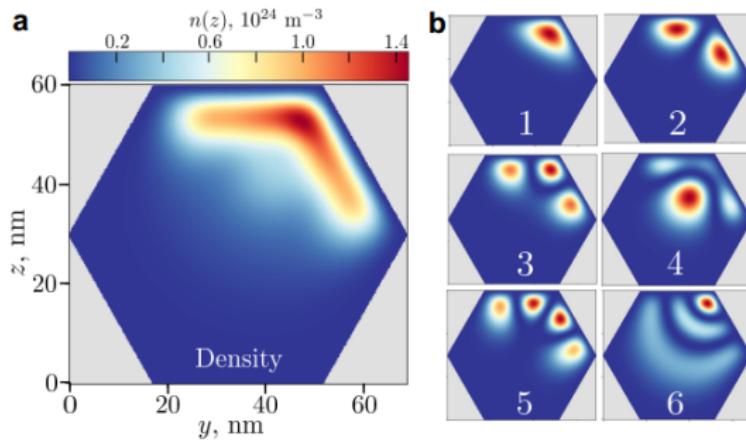


FIG. 3. (a) Electronic density in the cross-sectional cut of the nanowire for  $V_g = 0$  obtained the Thomas Fermi approximation. (b) Square modulus of eigenstates of the wire in the normal state at  $B = 0$  with energies  $-0.096, -0.068, -0.052, -0.023, -0.021$ , and  $-0.006 \text{ eV}$  for panels 1 to 6 respectively.

## 4. Эффект близости

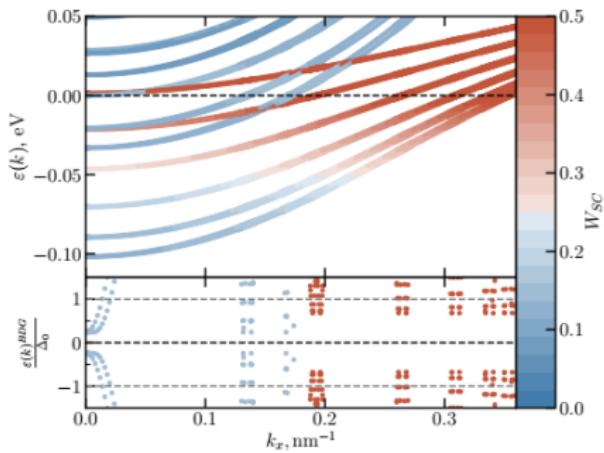


FIG. 6. Top: band structure in the normal state at  $V_g = -0.15$  V. Color indicates the weight of the state in the superconductor. Hybridization between states is seen by the changing color of the subbands. Bottom: band structure of the system in the superconducting state. The induced gap in each subband depends on the hybridization to the superconductor. It can clearly be seen that bands with stronger hybridization (red colors) are characterized by a larger induced gap. Here we used the same parameters as in Fig. 5(a) and (d).

## 4. Эффект близости (наведенная щель)

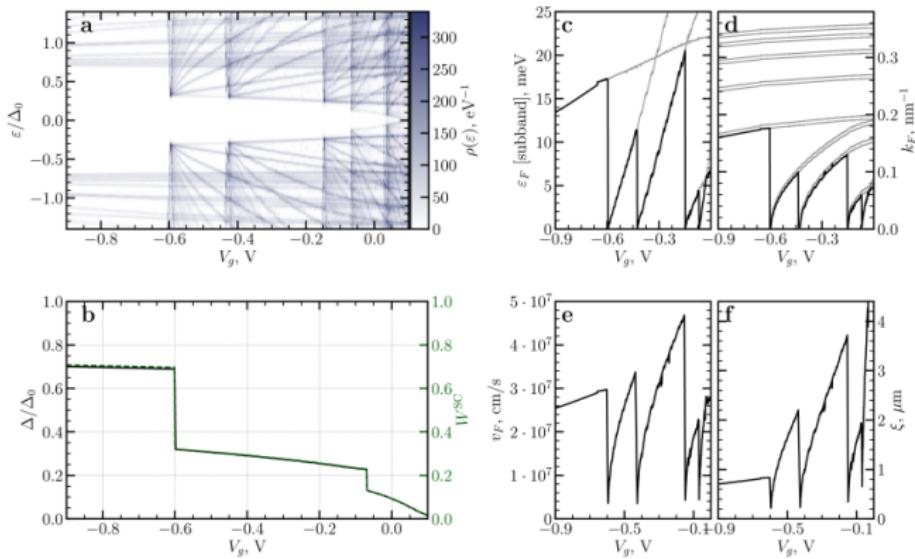


FIG. 7. Characterization of the superconducting state at  $B = 0$ , i.e. in the trivial  $s$ -wave superconducting phase, as a function of the gate voltage  $V_g$ : (a) Density of states, (b) induced gap (black) and weight in the superconductor (green), (c) Fermi energy, (d) Fermi momentum, (e) Fermi velocity and (f) SC coherence length. The discontinuities correspond to transitions where bands are driven below the chemical potential and thus become occupied (for an illustration, consider the transition between panels (a) and (c) of Fig. 8). In panel (b), the correspondence between the magnitude of the induced gap and the hybridization between SM and SC (as measured by the  $W_{SC}$ ) is clearly shown. In panels (c) and (d), all bands are shown in grey, while the occupied band closest to the Fermi energy is highlighted in black.

Спасибо за внимание!