Parity effect in the physics of superconductivity

Vadimov Vasiliy

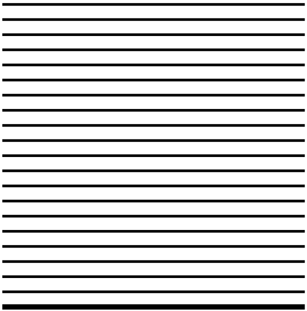
Institute for Microstructure Physics RAS, Nizhny Novgorod

November 1, 2018

Outline

- Parity effect in superconductors
- Single electron transistors with the superconducting isle
- Finite temperatures
- Superconducting ring with Josephson junction
- Ground state parity switching

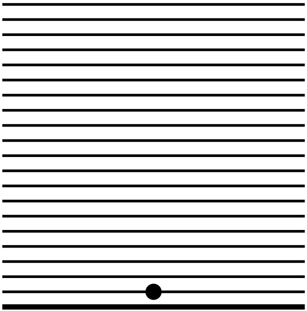
Normal metal (T=0)



Ground state

$$F = E - \mu N - TS$$

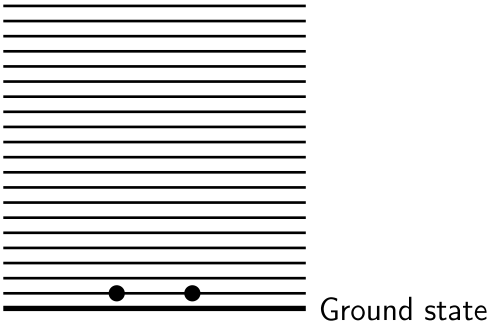
Normal metal (T=0)



$$F = E - \mu N - TS$$

$$\delta F = \frac{\partial \mu}{\partial N} = \frac{1}{\nu V}$$

Normal metal (T=0)



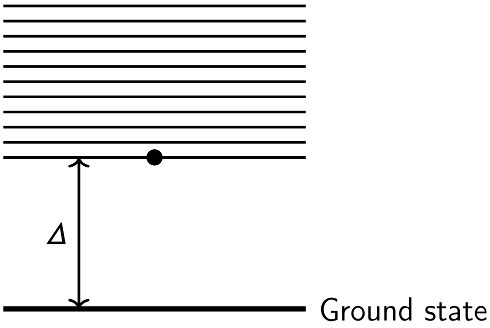
$$F = E - \mu N - TS$$

$$\delta F = 2\frac{\partial \mu}{\partial N} = \frac{2}{\nu V}$$

Superconductor (T=0)

$$F = E - \mu N - TS$$

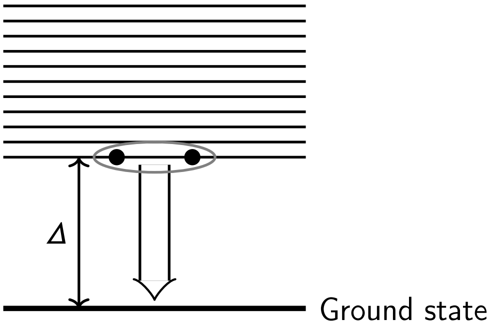
Superconductor (T=0)



$$F = E - \mu N - TS$$

$$\delta F = \Delta + \frac{\partial \mu}{\partial N}$$

Superconductor (T=0)

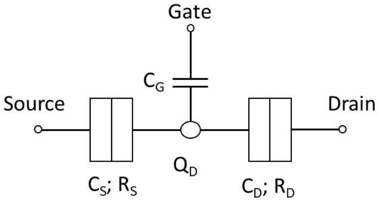


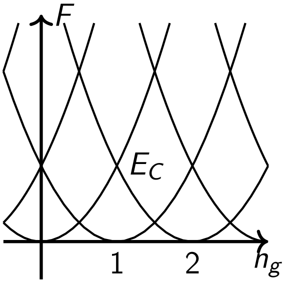
$$F = E - \mu N - TS$$

$$\delta F = 2\frac{\partial \mu}{\partial N}$$

Odd and even states have different energy!

Single electron transistor (SET)



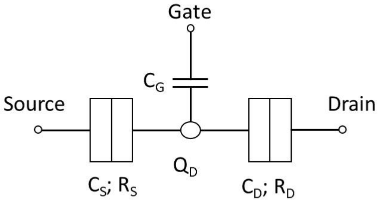


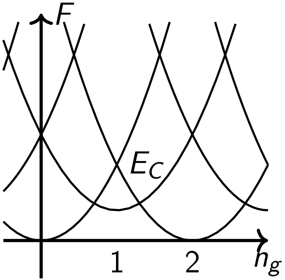
$$C_\Sigma = C_g + C_s + C_d$$

$$F = \frac{(V_g C_g - ne)^2}{2C_\Sigma} = E_C(n - n_g)^2$$

Normal isle

Single electron transistor (SET)



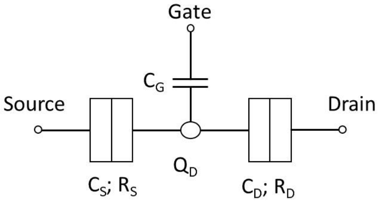


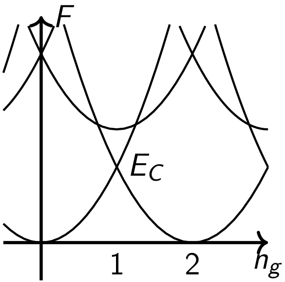
$$C_\Sigma = C_g + C_s + C_d$$

$$F = \frac{(V_g C_g - ne)^2}{2C_\Sigma} = E_C(n - n_g)^2$$

SC isle $F_o - F_e < E_c$

Single electron transistor (SET)



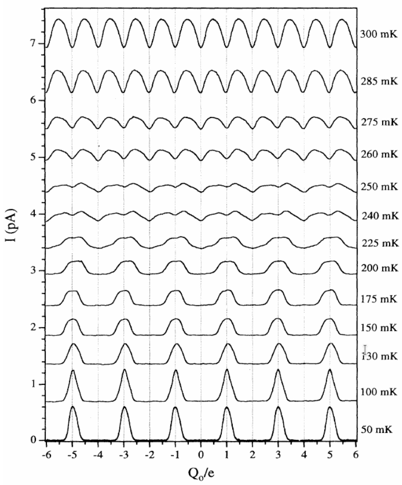


$$C_\Sigma = C_g + C_s + C_d$$

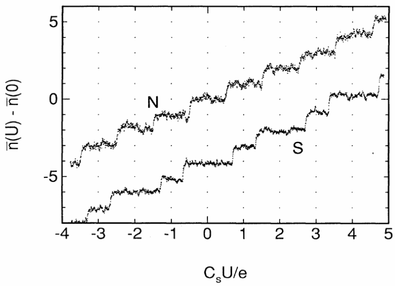
$$F = \frac{(V_g C_g - ne)^2}{2C_\Sigma} = E_C (n - n_g)^2$$

SC isle $F_o - F_e > E_c$

Experiments

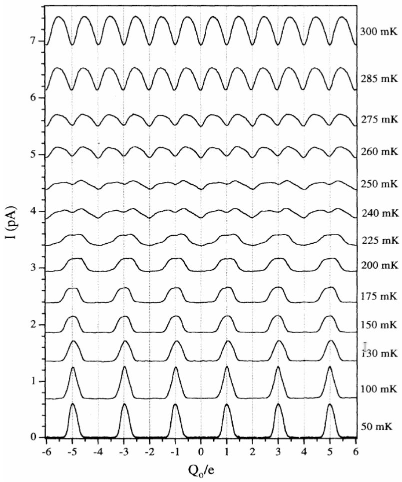


M. Tinkham, et. al., Phys. Rev. B **51**, 12649 (1994)

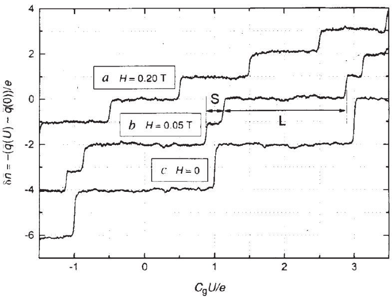


$T = 25\ mK$
P. Lafarge, et. al., Phys. Rev. Lett. **70**, 994 (1993)

Experiments

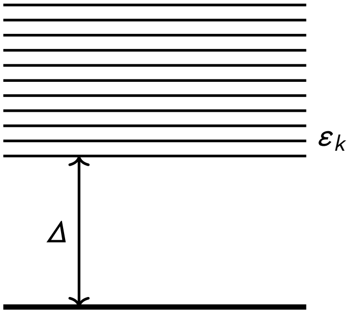


M. Tinkham, et. al., Phys. Rev. B **51**, 12649 (1994)



T = 28 *mK*
P. Lafarge, et. al., Letters to Nature **365**, 442 (1993)

Finite temperature

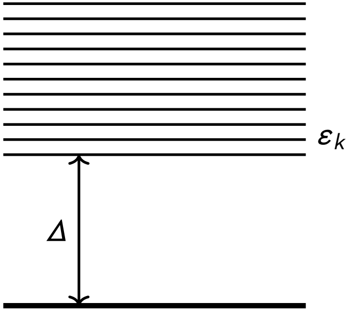


The ground state is supposed to be even

$$Z = e^{-E_0/T} \prod_k \left(1 + e^{-\varepsilon_k/T}\right)$$

$$Z_{e(o)} = \frac{e^{-E_0/T}}{2} \left[\prod_k \left(1 + e^{-\varepsilon_k/T}\right) \pm \prod_k \left(1 - e^{-\varepsilon_k/T}\right) \right]$$

Finite temperature



The ground state is supposed to be even

$$F_o - F_e \approx \Delta - T \ln \left(\nu V \sqrt{8\pi \Delta T} \right)$$

$$T_* \sim \frac{\Delta}{\ln(\nu V \Delta)} \ll \Delta$$

Finite temperature

Hamiltonian of the interacting electrons:

$$\hat{H} = \sum_{k\sigma} \xi_k \hat{a}^{\dagger}_{k\sigma} \hat{a}_{k\sigma} - \sum_{kk'} V_{kk'} \hat{a}^{\dagger}_{k\uparrow} \hat{a}^{\dagger}_{\bar{k}\downarrow} \hat{a}_{\bar{k}'\downarrow} \hat{a}_{k'\uparrow}$$

Bogoliubov transform:

$$\hat{\gamma}_{k\uparrow} = u_k \hat{a}_{k\uparrow} - v^*_k \hat{a}^{\dagger}_{\bar{k}\downarrow}$$

$$\hat{\gamma}_{k\downarrow} = u_k \hat{a}_{k\downarrow} + v^*_k \hat{a}^{\dagger}_{\bar{k}\uparrow}$$

$$\hat{H}_k = \sum_{k\sigma} \varepsilon_k \hat{\gamma}^{\dagger}_{k\sigma} \hat{\gamma}_{k\sigma} + \hat{H}_r = \hat{H}_q + \hat{H}_r$$

Finite temperature

Approximation for the density matrix is

$$\hat{\rho}_{e(o)} = \frac{\left(1 \pm e^{i\pi\hat{N}}\right) e^{-\hat{H}_q/T}}{\mathrm{Tr}\left(1 \pm e^{i\pi\hat{N}}\right) e^{-\hat{H}_q/T}} \; .$$

The free energy

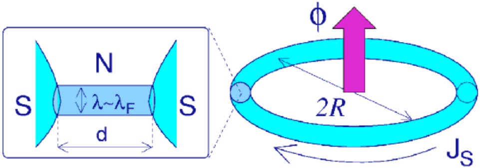
$$F_{e(o)} = \mathrm{Tr}\left(\hat{\rho}_{e(o)}\hat{H} + T\hat{\rho}_{e(o)} \ln \hat{\rho}_{e(o)}\right)$$

should me minimized over u_k, v_k and ε_k.

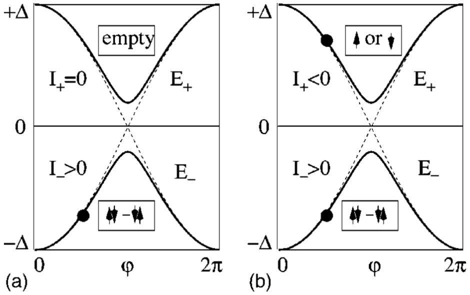
Corrections to Δ and the spectrum have order of $\left(\nu V\sqrt{\Delta T}\right)^{-1}$

Janko, et. al., Phys. Rev. B **50**, 1152 (1994)

Parity effect in Josephson junctions

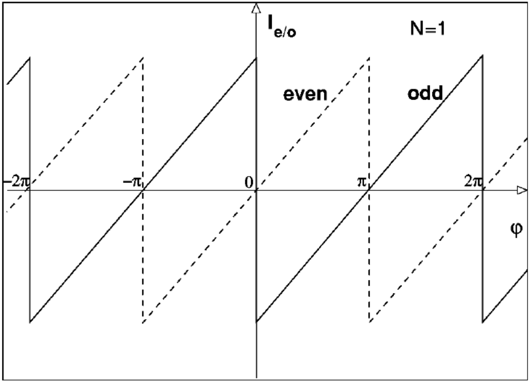


Quantum point contact



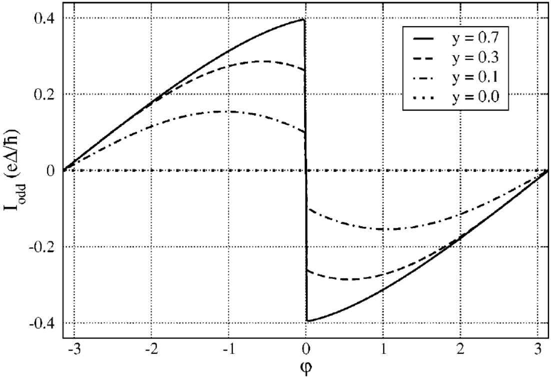
Spectrum of Andreev states

Parity effect in Josephson junctions



Current-phase dependence for the even/odd parity, $d/\xi \gg 1$.

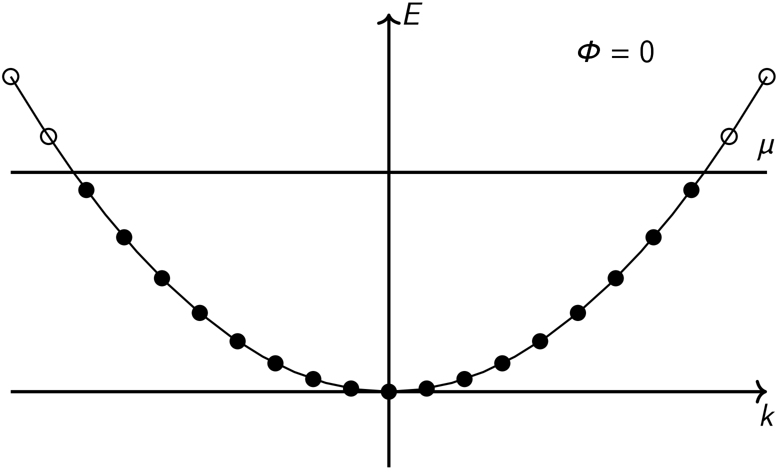
Parity effect in Josephson junctions



Current-phase dependence for the odd parity, $y = d/\xi \gg 1$.

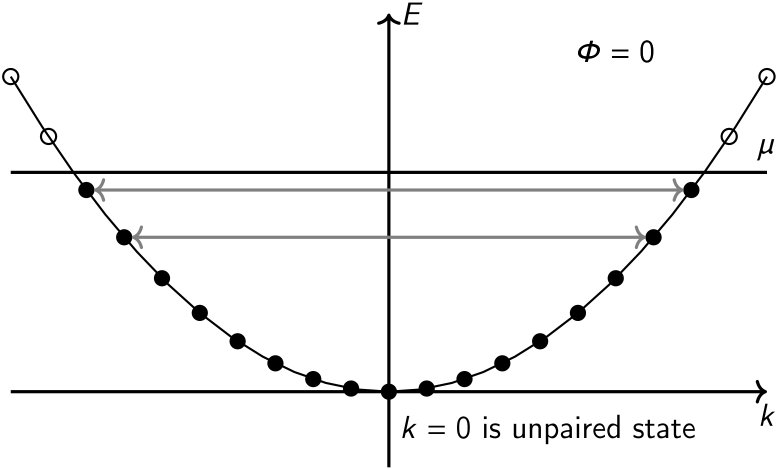
Ground state parity switching

1D p-wave superconducting ring. "Spinless" fermions (strong Zeeman interaction)



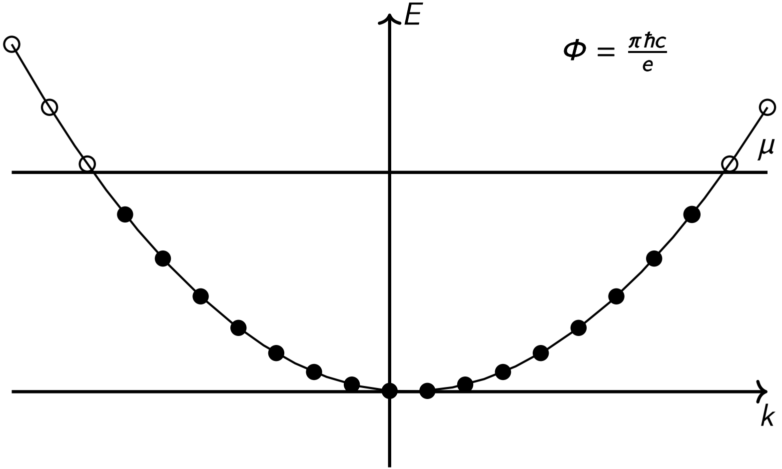
Ground state parity switching

1D p-wave superconducting ring. "Spinless" fermions (strong Zeeman interaction)



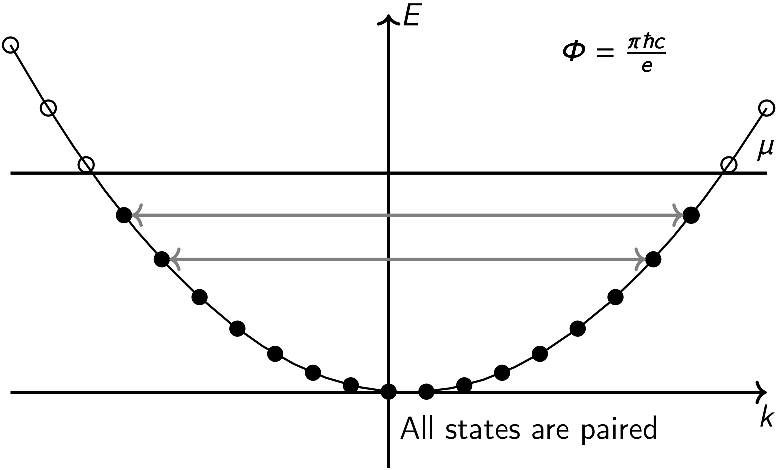
Ground state parity switching

1D p-wave superconducting ring. “Spinless” fermions (strong Zeeman interaction)

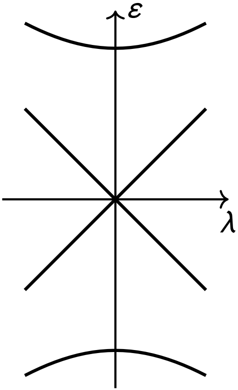


Ground state parity switching

1D p-wave superconducting ring. “Spinless” fermions (strong Zeeman interaction)



Ground state parity switching



BdG spectrum

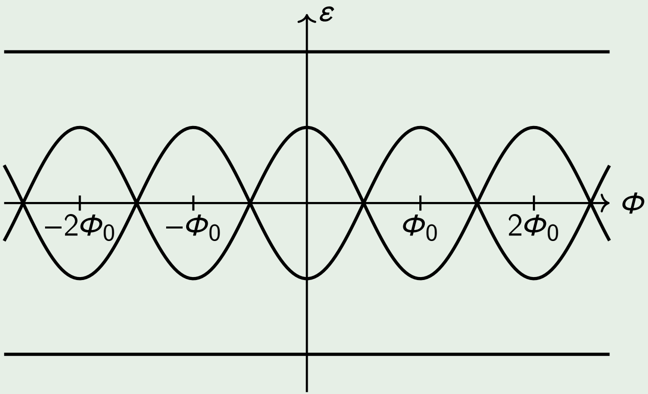
Parametric Hamiltonian

$$\hat{H}(\lambda) \rightarrow \varepsilon_n(\lambda)$$

Parity of the ground state switches at the zero crossing $\varepsilon_{n_0}(\lambda) = 0$, e.g. at $\lambda = 0$

Examples

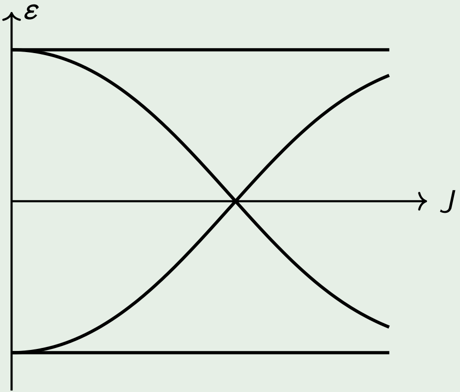
1D ring of *p*-wave superconductor with a Josephson junction



H. J. Kwon, et. al., Eur. Phys. J. B **37**, 349 (2004)

Examples

Yu–Shiba–Rusinov state in *s*-wave superconductor



Thank you for attention!